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ABSTRACT

This report is the initial overall summary of a comprehensive analysis of children's logical concept attainments and memoric abilities and deals exclusively with the initial and second year's assessment data. The focal point for this normative investigation is the concrete operations period spanning the years of middle childhood. The measurement battery consisted of logical concept tasks assessing classificatory abilities (groupements, I to IV, dichotomous sorting, some-all understanding, class inclusion, and combinatorial reasoning); relational abilities (groupements V to VIII, transitive inference--length and weight task formats, seriation--serial ordering, additive seriation, and serial correspondence); number concepts (cardinality, conservation and weight (identity and equivalence task formats); and three memory ability measures (forward word and digit series, and visual orientation memory). All the tasks were individually administered. It was concluded that the best characterization of the concrete operations period is a complex interrelationship pattern which is multidimensional in nature. The transition from preoperational forms of logical reasoning to a full mastery of the concepts derived from the logical groupements appears not to be a punctuate, discontinuous phenomenon with uniform progress for the various task settings. (RC)

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an initial analysis of concrete operations task performances and memory variables for children aged 5 to 13 years

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WISCONSIN RESEARCH AND DEVELOPMENT CENTER FOR COGNITIVE LEARNING

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Technical Report No. 371

AN INITIAL ANALYSIS OF CONCRETE OPERATIONS TASK PERFORMANCES AND MEMORY VARIABLES FOR CHILDREN AGED 5 TO 13 YEARS

by

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Report from the Project on
Children's Learning and Development

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TABLE OF CONTENTS

Chapter		Page
	Acknowledgments	iv
	List of Tables	vii
	List of Figures	xiii
	Abstract	xv
I	Introduction	1
II	Method	9
	General Assessment Procedures	9
	Subject Sample Descriptions	10
	Description of Task Battery	10
III	Results	19
	Preliminary Considerations	19
	Primary Analyses	23
IV	Discussion	86
	References	95
	Appendix	
	Intercorrelations Among Initial and Second Year Performances	101

LIST OF TABLES

Table		Page
1	Sampling Design for the Longitudinal Study	11
2	Distribution by Grade, Sex, Mean Age, and Age Range (in Years and Months) of the First Assessment Year Subject Sample	12
3	Distribution by Grade, Sex, Mean Age, and Age Range (in Years and Months) of the Second Assessment Year Longitudinal and Testing Control Subject Samples.	13
4	Hoyt Reliabilities for Various *Groupement* Subscales Combining Kindergarten, Third Grade, and Sixth Grade (N = 180) (Initial Assessment Year's Results). . .	20
5	Hoyt Reliabilities for Various Conservation Subscales Combining Kindergarten and Third Grade (N = 120) (Initial Assessment Year's Results).	21
6	Hoyt Reliabilities for the Seriation, Transitivity of Length, and Transitivity of Weight Subscales Combining Kindergarten and Third Grade (N = 120) (Initial Assessment Year's Results).	22
7	Hoyt Reliabilities for the Some-All and Cardinality Tasks Combining Kindergarten and Third Grade (N = 120) (Initial Assessment Year's Results).	22
8	Hoyt Reliabilities for Subscales of the Class Inclusion Task Combining Kindergarten, Third, and Sixth Grades (Initial Assessment Year's Results)	22
9	Means and Standard Deviations for the Four Class *Groupement* Tasks from the Initial Assessment Year's Results	24
10	Means and Standard Deviations for the Four Relations *Groupement* Tasks from the Initial Assessment Year's Results.	25

Table		Page
11	Means and Standard Deviations for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span Test, Auditory Sequential Memory Test, and Visual Orientation Memory from the Initial Assessment Year's Results	26
12	Means and Standard Deviations for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks from the Initial Assessment Year's Results.	28
13	Means and Standard Deviations for the Number Conservation, Length Conservation, and Weight Conservation Tasks from the Initial Assessment Year's Results	29
14	Means and Standard Deviations for the Seriation, Length Transitivity, and Weight Transitivity Tasks from the Initial Assessment Year's Results	30
15	Means and Standard Deviations for the Four Class *Groupement* Tasks Administered to the Longitudinal and Testing Control Subjects	35
16	Means and Standard Deviations for the Four Relations *Groupement* Tasks Administered to the Longitudinal and Testing Control Subjects	36
17	Means and Standard Deviations for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span Test, Auditory Sequential Memory Test, and Visual Orientation Memory for the Longitudinal and Testing Control Subjects	37
18	Means and Standard Deviations for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks for the Longitudinal and Testing Control Subjects	38
19	Means and Standard Deviations for the Seriation, Length Transitivity, and Weight Transitivity Tasks Administered to the Longitudinal and Testing Control Subjects	39
20	Means and Standard Deviations for the Number Conservation, Length Conservation, and Weight Conservation Tasks Administered to the Longitudinal and Testing Control Subjects	40

Table		Page
21	Factorial Variance Analyses Summary for the *Groupement* Tasks with Three Factors: Assessment Condition (Longitudinal Versus Testing Control Subjects), Grade Level (First, Fourth, and Seventh), and Sex. . . .	41
22	Factorial Variance Analyses Summary for Class Inclusion, Combinatorial Reasoning Task, Auditory Memory Span, Auditory Sequential Memory, and Visual Orientation Memory with Three Factors: Assessment Condition (Longitudinal Versus Testing Control Subjects), Grade Level (First, Fourth, and Seventh), and Sex.	42
23	Factorial Variance Analyses Summary for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks with Three Factors: Assessment Condition (Longitudinal Versus Testing Control Subjects), Grade Level (First and Fourth), and Sex . . .	43
24	Factorial Variance Analyses Summary for the Seriation, Length Transitivity, and Weight Transitivity Tasks with Three Factors: Assessment Condition (Longitudinal Versus Testing Control Subjects), Grade Level, and Sex (First and Fourth).	44
25	Factorial Variance Analyses Summary for the Number Conservation, Length Conservation, and Weight Conservation Tasks with Three Factors: Assessment Condition (Longitudinal Versus Testing Control Subjects), Grade Level (First and Fourth), and Sex.	45
26	Means and Standard Deviations for the Four Class *Groupement* Tasks for Subjects Assessed in Both First and Second Years	46
27	Means and Standard Deviations for the Four Relations *Groupement* Tasks for Subjects Assessed in Both First and Second Years	47
28	Means and Standard Deviations for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span Test, and Visual Orientation Memory for Subjects Assessed in Both First and Second Years.	48
29	Means and Standard Deviations for the Some-All, Dichotomous Sorting, Cardinality and Arithmetic Addition Tasks for Subjects Assessed in Both First and Second Years	49

Table		Page
30	Means and Standard Deviations for the Seriation, Length Transitivity, and Weight Transitivity Tasks for Subjects Assessed in Both First and Second Years.	50
31	Means and Standard Deviations for the Number Conservation, Length Conservation, and Weight Conservation Tasks for Subjects Assessed in Both First and Second Years	51
32	Factorial Variance Analyses Summary (Mixed Model) for the *Groupement* Tasks with Three Factors: Grade Level (Kindergarten-First/Third-Fourth/Sixth-Seventh), Sex, and Assessment Year	52
33	Factorial Variance Analyses Summary (Mixed Model) for Class Inclusion, Combinatorial Reasoning, Auditory Memory Span, Auditory Sequential Memory, and Visual Orientation Memory with Three Factors: Grade Level (Kindergarten-First/Third-Fourth/Sixth-Seventh), Sex, and Assessment Year	53
34	Factorial Variance Analyses Summary (Mixed Model) for the Some-All, Dichotomous Sorting, Cardinality, and Arithmetic Addition Tasks with Three Factors: Grade Level (Kindergarten-First/Third-Fourth), Sex, and Assessment Year.	54
35	Factorial Variance Analyses Summary (Mixed Model) for the Seriation, Transitivity of Length, and Transitivity of Weight Tasks with Three Factors: Grade Level (Kindergarten-First/Third-Fourth), Sex, and Assessment Year	55
36	Factorial Variance Analyses Summary for the Number Conservation, Length Conservation, and Weight Conservation Tasks with Three Factors: Grade Level (Kindergarten-First/Third-Fourth), Sex, and Assessment Year	56
37	Intercorrelations Among Tasks Administered to Year One Total Subject Sample (Combining Kindergarten, Third, and Sixth Grades)	61
38	Unrotated Principal Components Factor Matrix of Tasks Administered to Year One Total Subject Sample (Combining Kindergarten, Third, and Sixth Grades). . . .	62

Table		Page
39	Varimax Rotated Factor Matrix, Principal Components Procedure, of Tasks Administered to Year One Total Subject Sample (Combining Kindergarten, Third, and Sixth Grades)	64
40	Intercorrelations Between Initial and Second Year Performances for the Measures Administered to the Total Subject Sample (N = 156)	66
41	Intercorrelations Among Tasks Administered to Year Two Total Subject Sample (Combining First, Fourth, and Seventh Grades)	67
42	Unrotated Principal Components Factor Matrix of Tasks Administered to Year Two Total Subject Sample (Combining First, Fourth, and Seventh Grades)	69
43	Varimax Rotated Factor Matrix, Principal Components Procedure, of Tasks Administered to Year Two Full Sample (Combining First, Fourth, and Seventh Grades)	70
44	First and Second Assessment Year Intercorrelations Among the Memory Tasks and Combinatorial Reasoning, and Class Inclusion for the Third/Fourth, and Sixth/ Seventh Grade Subjects	72
45	Intercorrelations Among Tasks Administered to Year One Kindergarten and Third Grade Subject Sample	74
46	Unrotated Principal Components Factor Matrix of Complete Year One Assessment Battery Tasks (Combining Kindergarten and Third Grade)	77
47	Varimax Rotated Factor Matrix, Principal Components Procedure, of Complete Year One Assessment Battery Tasks (Combining Kindergarten and Third Grade)	78
48	Intercorrelations Between Initial and Second Year Performances for the Measures Administered to the Kindergarten/First and Third/Fourth Grade Subjects (N = 102)	80
49	Intercorrelations Among Tasks Administered to Year Two First and Fourth Grade Subject Sample	81
50	Unrotated Principal Components Factor Matrix of Complete Year Two Assessment Battery Tasks (Combining First and Fourth Grades)	83

Table		Page
51	Varimax Rotated Factor Matrix, Principal Components Procedure, of Complete Year Two Assessment Battery Tasks (Combining First and Fourth Grades).	84
52	Correlations and Reliability Ratios for Cross-Lag Analysis by Task and Year of Assessment.	91

LIST OF FIGURES

Figure		Page
1	Representation of Year One Task Performances in the Plane of Factors One and Three	63
2	Year One to Year Two Changes in Location of Piagetian Tasks in the Space Defined by *Groupements* and Memory Factors	71
3	Cross-Lag Year One and Year Two Correlations for the Orientation Memory and Combinatorial Reasoning Tasks	90
4	Corrected Year One and Year Two Correlations for The Orientation Memory and Combinatorial Reasoning Tasks	92

ABSTRACT

The initial and second year's performances of a sample of children aged 5 to 13 years on a representative series of concrete operations period logical reasoning tasks and memory measures are reported. These results represent the initial phase of a comprehensive longitudinal/cross-sectional assessment project conducted over a four-year time interval. Subject samples for the initial year (1973) included 30 males and 30 females at each of the following grade levels: kindergarten, third, and sixth grades. One hundred and fifty six of the original subjects were available for retesting in 1974. In addition, 24 testing control subjects were randomly selected from each age-grade cohort group and assessed in 1974 only. The measurement battery consisted of logical concept tasks assessing classificatory abilities (*groupements* I to IV, dichotomous sorting, some-all understanding, class inclusion, and combinatorial reasoning); relational abilities (*groupements* V to VIII, transitive inference [length and weight task formats], seriation [serial ordering, additive seriation, and serial correspondence]); number concepts (cardinality, conservation of number, arithmetic addition and subtraction); conservation of length and weight (identity and equivalence task formats); in addition to three memory ability measures (forward word and digit series, and visual orientation memory). All the tasks were individually administered.

Preliminary analyses indicated a generally satisfactory level of internal consistency reliability across the present task array and an absence of significant order of presentation or school location effects. Analysis of the initial year's performances indicated significant grade level main effects for all the measures except length transitivity. Sex distinctions were minimal (the single exception concerned significant female subjects' superiority on class inclusion understanding) and there was an absence of significant grade/sex interactions. Comparisons of relative item difficulties across tasks (e.g., transitive inference/conservation, some-all/class inclusion, identity conservation/equivalence conservation, and class *groupements*/relations *groupements*) essentially supported the results of previous investigations and may be viewed as contrary to the predictions of orthodox Piagetian stage theory (i.e., the unitary assumptions of the *structure d'ensemble* postulate). Additional in-depth analyses of the number, conservation, and *groupements* task arrays are also summarized.

Preliminary analysis of the second year's results and the initial/second year performance increments concerned longitudinal sample attrition and repeated testing effects. Comparisons of the initial year's performances

of the continuing subjects (N = 156) and the "dropout" subjects (N = 24) revealed few consistent distinctions. Comparisons of the second year's performances of the longitudinal subjects and testing control subjects indicated minimal repeated measurement effects were present. Direct comparisons (mixed model variance analyses) of the relative difficulties of the transitive inference, conservation, and class inclusion tasks for the two assessment years indicated the following developmental pattern: Transitive inference → conservation = class inclusion.

Dimensional analyses were also computed for the initial and second year's longitudinal sample performances. The initial year's total subject analysis (the sixth grade subjects were administered a subset of tasks consisting of the *groupements*, class inclusion, combinatorial reasoning, and three memory tasks) resulted in three factors; i.e., (1) relations, (2) classes, and (3) memory. The second year's performance patterns were similar (i.e., three primary factors were found) although certain tasks, most notably class inclusion and combinatorial reasoning, demonstrated differential factor loadings in comparison to their initial assessment values with the class *groupements* beginning to explain Piagetian task-related variance, as had not been the case in the initial year. Dimensional analyses for the complete assessment battery (kindergarten/first and third/fourth grade subjects) indicated four rotated factors, i.e., (1) conservation, (2) auditory memory, (3) relations, and (4) classes. The corresponding second assessment year analysis yielded five factors, i.e., (1) conservation, (2) number concepts, (3) classes, (4) memory, and (5) relations-transitivity. Thus, the complete assessment battery analysis indicates increasing differentiation among the tasks from year one to year two.

It was concluded that the best characterization of the concrete operations period is a complex interrelationship pattern which is multidimensional in nature. The transition from preoperational forms of logical reasoning to a full mastery of the concepts derived from the logical *groupements* appears not to be a punctate, discontinuous phenomenon with uniform progress for the various task settings.

I

INTRODUCTION

The present technical report is the initial overall summary of a comprehensive analysis of childrens' logical concept attainments and memoric abilities. The general rationale, sampling and assessment design, and specific objectives of the larger study have been reported previously (Hooper & Klausmeier, 1973). The present report will deal exclusively with the initial and second years' assessment data. A number of previous publications (i.e., Brainerd & Hooper, 1975; Dihoff, 1975, 1976; Gonchar, 1975; Hooper, Brainerd, & Sipple, 1975; Hooper & Toniolo, 1976; Klausmeier, Hooper, & Sipple, 1976; Toniolo & Hooper, 1975) have dealt with various specialized aspects of this four-year longitudinal study.

The focal point for the present normative investigation is the concrete operations period spanning the years of middle childhood. While this developmental interval has been the subject of a great many cross-sectional investigations (cf. Flavell, 1970; Hooper, Goldman, Storck, & Burke, 1971; Hooper, Sipple, Goldman, & Swinton, 1974; Sigel & Hooper, 1968) very few studies have employed a longitudinal assessment design. As is well known, the traditional cross-sectional research design is subject to notable deficiencies and does not properly represent a true experimental design (Baltes, 1968; Campbell & Stanley, 1963; Schaie, 1965, 1973; Wohwill, 1970, 1973). Longitudinal assessment designs also have potential confounding factors present (e.g., selective sampling and survival effects, repeated measurement factors and problems of external validity) but these designs are generally considered to be superior. In particular, longitudinal assessments (in addition to experimental modification studies) are the only strategies appropriate to the identification of developmental sequences and the interpatterning among developmental variables over time. A longitudinal design with specific provision for repeated measurement effects evaluation was employed in the present investigation.

As mentioned above, there is a considerable body of original and derived research which has dealt with concrete operations period functioning. The original Genevan sources include the following: Inhelder and Piaget (1958, 1964); Piaget (1942, 1970a, 1970b, 1972); Piaget and Inhelder (1962, 1956);

Piaget, Inhelder, and Szeminska, (1960); Piaget and Szeminska (1952). Recent reviews of the secondary research include Flavell (1970); Hooper, Sipple, Goldman, and Swinton (1974); and Klausmeier and Hooper (1974). During this general age interval, cognitive development as exemplified by the logical concepts dealing with classes, relations, number and their infralogical counterparts, is said to be in isomorphic correspondence to a logical model. Piaget (1942, 1972) employs the term *groupement* to indicate the structural unit characteristic of the middle childhood age interval. This term signifies "that these structures are syntheses of the common group and lattice structures of abstract algebra [Brainerd, 1972, p. 4]."

As has been stated previously:

> Operativity in the strict sense only becomes possible during the middle childhood years. It is during this time that the child's cognitive activities become organized into coherent and integrated systems. The term operation is used to refer to these systems both as something the child has at his disposal (i.e., activities which the child is capable of performing) and as something the child typically does (Furth, 1969). Concrete operations are internalized representations of coordinated systems called groupings or *groupements*. Viewed as something the child does, operations include the logico-mathematical activities of adding, subtracting, dividing, setting terms in correspondences as in the cases of classes and relations, and the infralogical activities (anchored to a specific spatio-temporal context) which deal with quantity, measurement, space, and time concepts (cf. Flavell, 1963, p. 166). Viewed as something the child has, operations must be considered as parts of a higher-order structural organization. As Flavell has stated: "One cannot really grasp the concept of class without understanding what a classification system entails, because the single class is only an abstraction from the total system. This is the central meaning of Piaget's holism in the domain of cognitive operations: the isolated operation can never be the proper unit of analysis, because it gains all its meaning from the system of which it is a part. A given operation, put into concrete effect in the here and now, always presupposes a structured system which includes other, related operations, for the moment latent and inactive but always potentially actualizable themselves and, above all, always a force governing the form and character of the operation which is momentarily on stage" (Flavell, 1963, p. 167).

> It is these supraordinate structures, the logical operations *groupements*, which are said to interrelate and subsume all directed thought during the middle childhood period. Four of the *groupements* are concerned with logical classes, and four complementary cases deal with logical relations. The most important of these include primary addition of classes (as evidenced, for example, in the class inclusion problem), bi-univocal multiplication of classes (as shown in matrix completion or class intersection problems), addition of asymmetrical relations (as evidenced in seriation and transitive inference tasks, e.g., the conclusion that A must be less than C, on some dimension if shown that A < B and B < C), and bi-univocal multiplication of relations (as assessed, for example, in the serial correspondence and double seriation matrix tasks). [Hooper & Klausmeier, 1973, pp. 14-15]

The present investigation is directed toward a formidable research objective: the empirical assessment of the behavioral consequences of Piaget's logical model of logical reasoning as exemplified by the concrete operations *groupements*. Brainerd has stated the argument for reduction of the myriad specific concrete operations surface structure tasks as follows:

> Piaget contends that a given structure S is engendered by either a cognitive operation and its inverse--if S is one of the four "class" *groupements*--or by a cognitive operation and its reciprocal--if S is one of the four "relational" *groupements*. In theory, the notion of "operation" can here be taken to mean almost any internalizable property of overt action. In practice, however, the notion may be restricted to the set-theoretic operations of union and intersection and the arithmetic operations of addition and multiplication. Thus each class *groupement* owes its existence to the fact that the cognizer is capable of simultaneously uniting and subtracting classes, while each relational *groupement* owes its existence to the fact that the cognizer is instantaneously aware of the reciprocal implications of the equivalence and difference relations produced by arithmetic addition and multiplication. To illustrate, the capacity simultaneously to add two subordinate classes to yield a superordinate class and to subtract one subordinate class from the superordinate class to yield the remaining subordinate class engenders a certain class *groupement* structure. Also to illustrate, the knowledge that A < B and B < C simultaneously implies both A < C and C > A engenders a certain relational structure. Although one may quibble with these claims about the origins of *groupement* structures on theoretical grounds, there is no denying their obvious empirical implication:

> If one wishes to conduct research on these structures, then one most certainly will have to focus on the unions and intersections of hierarchical classes and their inverses--on the one hand--and on the addition-multiplication of equivalence-difference relations and their reciprocals--on the other. [Brainerd, 1972, p. 5]

In view of the fact that the logical *groupements* "performances" are theoretically predicted to emerge and evidence final mastery in parallel or concomitant fashion, developmental synchrony is anticipated for all the classificatory and relational concept domains. This prediction follows directly from the orthodox Piagetian conception of the logical structure construct and underlies the assumption of stages as *structure d'ensemble* (cf. Brainerd, 1972; Flavell, 1970, 1971; Hooper, 1973; Piaget, 1972; Pinard & Laurendeau, 1969) Relatively consistent performances (low intra-individual variability) should be observed for concept tasks (equated for general item difficulty) designed to assess behaviors derived from common structural components. Thus, equivalent tasks assessing class, relations, and natural number concepts should evidence similar relative item difficulties (cross-sectional data) and congruent developmental acquisition patterns (longitudinal data) according to orthodox Piagetian theory.

An antithetical corollary to the stage-correspondence assumption is within-stage sequential linkages, i.e., the observation that mastery of certain less complex tasks is a genetic (and perhaps a logical) precursor of subsequent higher order abilities. Certain empirically demonstrated cognitive sequences (and, hence, asynchronies) are difficult to reconcile within conventional Piagetian systems (cf. Brainerd, 1972, 1973a, 1973b, 1974b; Brainerd & Fraser, 1975; Flavell, 1971, 1972; Hooper & Toniolo, 1976; Weinreb & Brainerd, 1975).

Perceptual theorists (e.g., Neisser, 1967) have traced the development of short-term memory as a maturational increase in the capacity of a postulated storage register. Drawing an analogy from information theory, McLaughlin (1963) suggested a correspondence between Piaget's stages of reasoning and the number of bits of information available in a central processor (see also Neimark, 1970; and Simon, 1962). This approach has been advocated by Klahr and Wallace (1970, 1972) and Pascual-Leone (1970) who have presented detailed analyses of the steps required to solve specific Piagetian tasks, predicting relative difficulty from the degree of memory load that such analyses reveal. These approaches represent a view of memory as a developmentally-determined upper limit on the complexity of possible cognitive operations, and imply that individual differences in short-term memory will contribute error variance to attempts to examine the relative difficulty and sequence of acquisition of structures dealing with relational and classificatory concepts. To the extent that specific tasks seeking to embody these structures vary in incidental memory demands or salience of features to be

encoded into memory, we consider these memory factors to be covariates, the effects of which we wish to minimize to better understand other variables more central to the model.

A small number of empirical studies have investigated the relationship of memory variables and logical concept functioning (e.g., Furth, Ross, & Youniss, 1974; Liben, 1974, 1975; Neimark, Slotnick, & Ulrich, 1971; Roodin & Gruen, 1970; Rybash, Roodin, & Sullivan, 1975). An information processing performance model for such Piagetian tasks as conservation of number, transitivity of length, or class inclusion has not yet been completely developed, although impressive strides in this direction have been made (Klahr & Wallace, 1972). It is clear, however, that such a model will include logical operators as typified by the *groupement* structures in the present study, working memory or computing registers, and a specification of the encoding and organization of material in long-term memory.

In contrast, a true Piagetian model of development would also make provision for the action of the logical operators on the contents and structure of long-term memory, representing an active restructuring of the entire system (cf. Liben, 1976, Piaget, 1968; Piaget & Inhelder, 1973). This more general Piagetian model would go beyond analogies to current computers, in which operations are performed only on material in a relatively restricted central computing register, data and results being read in and out of a fixed-structure memory, to model qualitative changes in organization and operation of the structure of the memory itself with increasing maturity.

A descriptive-correlational study cannot test such competing cognitive models, or lead to descriptions of the interrelations among their components. It can, however, offer evidence concerning the degree to which such components as the class and relational *groupement* and memory variables as assessed in this study must be considered in the construction of such models. To the extent that patterns of relationship of logical and memoric variables different from the Piagetian are not uncovered, a simpler associationistic or S-R learning model could suffice to describe growth of performance over time.

The present data enable us to examine the patterns of relationships among concrete operations tasks, direct measures of postulated cognitive operations, and memory abilities. This makes possible the analyses of the degree to which these relationships are congruent with analyses that imply differing involvement of the underlying cognitive and memory variables in the performance of various concrete tasks. While this examination is complicated by the existence of shared method variance among some subsets of tasks, ceiling effects, and differences in reliability of measurement of the various constructs, certain comparisons can be made.

One important qualification must be noted, however. To the extent that the more general Piagetian model applies, in which memory is a product, and not merely a precursor of cognitive

development, our tendency to treat memory variables as inputs or covariates is an oversimplification. This could lead to inflated estimates of the causal role of figural memory in these performances, if a more complete model in which the rather strong observed relationship between figural memory and concrete operational tasks was treated as one of mutual causation is in fact appropriate. Work on the specification of such a path model is underway, and will be the basis of a future analysis (cf. Labouvie, 1974).

The approach to the structural analysis of the tasks adopted in this discussion is one of seeking parsimonious description of a number of inter-relationships in relatively few dimensions. It is not deemed advisable to invest these dimensions with psychological reality or to reify them with such terms as "underlying structure," in the absence of convergent evidence external to these particular data. This caution grows from a recognition that the results of components analysis of the covariance of tasks depend critically on the distribution of tasks in the test battery, and on the range of abilities in the population to which the battery was administered.

In the present study, for example, the inclusion of four quite similar conservation tasks essentially guarantees a conservation factor, but leaves open the question of which other tasks will be found to covary with this factor. The influence of variance within the sample can be expected to lead to a strong first factor tapping all age-related tasks when scores from the full seven-year age span are analyzed, while analyses within years should reveal more differentiation into specific factors.

In the present study an attempt was made to devise an array of Piagetian logical reasoning tasks representative of the concrete operations period of development. The general concept domains included classificatory abilities (*groupements* I-IV, dichotomous sorting, some-all understanding, class inclusion, and combinatorial reasoning); relational abilities (*groupements* V-VIII, transitive inference [length and weight task formats], seriation [serial ordering, additive seriation, and serial correspondence]); number concepts (cardinality, conservation of number, arithmetic addition and subtraction); conservation of length and weight (identity and equivalence task formats); and three memory ability measures (forward word and digit series, and visual orientation memory). These measures were presented in three individually administered sessions to groups of preschool (conservation and transitive inference tasks only), kindergarten, third, and sixth grade children. The surviving subjects from the three oldest age-grade samples, in addition to a sample of matched cohort control subjects, were retested one year later. All of the children in the continuing longitudinal sample also received a series of concept tasks based

upon the concept learning and development model (Klausmeier, Ghatala, & Frayer, 1974; see also Hooper & Klausmeier, 1973). The results of the Piagetian/CLD concept interrelationship analyses are reported elsewhere (Klausmeier, Hooper, & Sipple, 1976).

II

METHOD

GENERAL ASSESSMENT PROCEDURES

As an integral step in the comprehensive, four year longitudinal study of concept development, representative samples of kindergarten, third grade, and sixth grade children were initially assessed in the spring of 1973 with a series of logical concept tasks based upon Piagetian theory, as well as with an arithmetic computation task, and selected measures of memoric abilities. The task array was designed to ensure a certain minimal degree of across-subject and across-task standardization while maintaining essential comparability with the earlier Piagetian concrete operations concept assessment research. All measures were individually administered and an attempt was made to retain the desirable flexibility features of the *méthode clinique*. Thus, following the establishment of essential rapport with the subject, the task settings were administered in a relaxed, game-like atmosphere intended to be nonanxiety producing. No explicit corrective reinforcement was introduced except in the initial warm-up sessions. Questions were restated if the child's responses were ambiguous or if an obvious misunderstanding of the criterial question was evident.

In the initial assessment year, the kindergarten and third grade subjects were presented the complete battery in three separate sessions. The tasks in the first session included (a) seriation, (b) conservation of number, (c) arithmetic computation, (d) cardinality, and (e) dichotomous sorting presented in a 30 to 40 minute period to each subject in one of four orders.[1]

The second session tasks were administered in a 35 to 40 minute sitting in the following order: visual orientation memory, auditory memory span, auditory sequential memory, *groupements*, combinatorial reasoning, and some-all. Tasks in the third and final session included transitivity, conservation, and class inclusion.[2] The transitive

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[1]The order of each pair of tasks was unique. Order 1: E, D, A, B, C; Order 2: A, C, D, B, E; Order 3: E, B, D, C, A; Order 4: C, B, A, D, E.

[2]An additional task format using drawings of children was employed in the first year only. Inspection revealed no significant differences in difficulty between this measure and the measure using geometric figures. Therefore, the former task was eliminated from subsequent administrations.

inference, identity conservation, and equivalence conservation tasks were presented in one of six counterbalanced orders. In addition, the length case always preceded the weight case within the transitivity, identity, and equivalence task formats. The class inclusion measure was presented last in the 15 to 20 minute session.

An abbreviated version of the assessment battery was administered in a single session to the sixth grade subjects in a 20 to 30 minute session in the following fixed order of presentation: visual orientation memory, auditory memory span, auditory sequential memory, *groupements*, combinatorial reasoning, and class inclusion.

As part of the continuing longitudinal study, the same kindergarten, third, and sixth grade children who were available for testing 12 months later as first, fourth, and seventh graders were administered the same tasks as in the first year. In addition, control groups of children were assessed in the spring of 1974 in order to determine the possible effects of longitudinal testing. The percentages of testing control subjects and longitudinal subjects tested by each administrator were as close to equal as possible. Whenever possible, each subject administered the tasks in the first and second years of the study was tested by the same test administrator in both years.

SUBJECT SAMPLE DESCRIPTIONS

One hundred and eighty school children enrolled in the Beloit, Wisconsin, public school system participated in the first-year assessments in the spring of 1973. Table 1 presents the general sampling design for the four-year longitudinal study. The children were randomly drawn from kindergarten, third, and sixth grade classrooms of four different elementary schools. These schools were selected in accord with the representative socioeconomic, ethnic, and general demographic classifications found in the city of Beloit. Table 2 presents a summary description, according to age and sex, of the 30 boys and 30 girls at each grade who took part in the initial year of the study. The age range of children within each grade varied from 12 to 17 months.

Of the 180 children who took part in the first year of the study, 156 children were available for longitudinal assessment 12 months later in 1974. Attrition was primarily due to entry into parochial schools or moving out of the city. The numbers of children retested in the second year at each grade group--first, fourth, and seventh grades--are given in Table 3. In the second year assessment, the children of the two lower grades were enrolled in 10 different elementary schools and seventh graders were enrolled in 2 different junior high schools.

DESCRIPTION OF TASK BATTERY

Detailed task descriptions and sources as well as complete administration procedures with associated scoring protocols are given in A Representative Series of Piagetian Concrete Operations Tasks (Hooper,

TABLE 1

SAMPLING DESIGN FOR THE LONGITUDINAL STUDY

(Underlined entries are approximate mean ages and arrows represent continuity of subgroups)

	Time of Measurement			
Cohort	1973	1974	1975	1976
1967	6 (N = 60) [Kindergarten]	→ 7 (N = 48)	→ 8 (N = 46)	→ 9 (N = 41)
1967		7 (N = 24)		
1967			8 (N = 20)	
1967				
1964	9 (N = 60) [Third Grade]	→ 10 (N = 54)	→ 11 (N = 50)	→ 12 (N = 47)
1964		10 (N = 24)		
1964			11 (N = 22)	
1964				
1961	12 (N = 60 [Sixth Grade]	→ 13 (N = 54)	→ 14 (N = 52)	→ 15 (N = 46)
1961		13 (N = 24)		
1961			14 (N = 24)	
1961				

TABLE 2

DISTRIBUTION BY GRADE, SEX, MEAN AGE, AND AGE RANGE (IN YEARS AND MONTHS) OF THE FIRST ASSESSMENT YEAR SUBJECT SAMPLE

Grade	Subjects	Males	Females	Mean Age	Age Range
K	60	30	30	5-10	5-3 to 6-3
3	60	30	30	8-9	8-3 to 9-8
6	60	30	30	11-10	11-3 to 12-8

TABLE 3

DISTRIBUTION BY GRADE, SEX, MEAN AGE, AND AGE RANGE (IN YEARS AND MONTHS) OF THE SECOND ASSESSMENT YEAR LONGITUDINAL AND TESTING CONTROL SUBJECT SAMPLES

Grade	Assessment Condition	Subjects	Males	Females	Mean Age	Age Range
1	Longitudinal	48	26	22	6-10	6-3 to 7-3
	Testing Control	24	12	12	6-10	6-4 to 7-4
4	Longitudinal	54	26	28	9-10	9-3 to 10-8
	Testing Control	24	12	12	9-10	9-4 to 10-6
7	Longitudinal	54	26	28	12-10	12-3 to 13-8
	Testing Control	24	12	12	12-11	12-6 to 13-8

Brainerd, & Sipple, 1975). This section will provide an abbreviated version of the descriptions presented in that paper.

First Assessment Session Tasks

The five separate seriation task items measured (1) serial construction, (2) serial addition, (3) serial correspondence in a matching case format, (4) serial correspondence in extension and compression case formats, and (5) serial correspondence in a scrambled case format. The theoretical score range for the seriation task was 0 to 5.

The conservation of number measure included (1) three items each which required predicting the equality of two sets of equal number under a verbal condition of altering one set, (2) three items of judgment of equality of the two sets after actual deformation of one set, and (3) one item of judgment of inequality of the two sets after one member of one set is removed. The conservation of number task score range was 0 to 7.

The arithmetic computation task consisted of 16 incomplete addition equation items presented in written form and read aloud to the subject. The score range was 0 to 16.

The cardinality task included the following five judgments: (1) the non-equivalence of number of two sets dispersed over equal lengths, (2) the non-equivalence of number of two sets dispersed over unequal lengths, (3) the equivalence of number of two sets dispersed over unequal lengths, (4) the equivalence or non-equivalence of number of two pairs of single digits presented verbally, and (5) the equivalence or non-equivalence of number of two sets dispersed over equal lengths and over unequal lengths. The score range was 0 to 5.

In the dichotomous sorting task, the S was asked to classify 22 pictorial stimuli, according to one of three dimensions, regroup the same stimuli on another of the dimensions, and finally regroup the same stimuli on the third dimension. The score range was 0 to 3.

Second Assessment Session Tasks

The visual orientation memory test (Swinton, 1973) assessed the S's ability to recognize and identify a two-dimensional geometric drawing from a pictorial array of the same figure in varying spatial orientations. There were four possible rotation or reflection choices for each of 20 items. The score range was 0 to 20.

The auditory memory span test (Wepman & Morency, 1973a) consisted of a forward word series with three trials of differing word sets at each level from two to six words (score range = 0 to 60). The auditory sequential memory test (Wepman & Morency, 1973b) consisted of a forward digit series (2 to 8 digits) with two sets of differing numbers at each level (score range = 0 to 70).

Groupement I: Primary Addition of Classes (Score Range = 0 to 8)

1. In reference to triangles, some blue, some yellow, some red,

and some both yellow and red, the S is asked whether the triangles of some specific color are the same (more) in number as all the triangles. The logical operation characterizing the psychological process is the composition (addition) of hierarchical primary classes A (triangles with yellow on them) + A' (triangles without yellow on them) = B (triangles). 4 trials.

2. In reference to the same stimuli, the S is asked whether there would be any members of the superordinate class remaining if all the members of the subordinate class were removed. The logical operation characterizing the psychological process is the inversion (subtraction) of hierarchical primary classes B (triangles) - A (triangles with yellow on them) = A' (triangles without yellow on them). 4 trials. (The above procedure in 1 and 2 is repeated with circles.)

Groupement II: Secondary Addition of Classes (Score Range = 0 to 8)

1. In reference to the same stimuli used in *Groupement* I, the S is asked whether the triangles of a different specific color are the same (more) in number as all the triangles. The logical operation characterizing the psychological process is the composition (addition) of hierarchical secondary classes A_2 (triangles with red on them) + A_2' (triangles without red on them) = B (triangles). 4 trials.
2. In reference to the same stimuli, the S is asked whether there would be any members of the superordinate class remaining if all the members of a subordinate class were removed. The logical operation characterizing the psychological process is the inversion (subtraction) of hierarchical secondary classes B (triangles) - A_2 (triangles with red on them) = A_2' (triangles without red on them). 4 trials. (The above procedure in 1 and 2 is repeated with circles.)

Groupement III: Bi-univocal Multiplication of Classes (Score Range = 0 to 8)

1. In reference to the same stimuli used for *Groupement* I and *Groupement* II, the S is asked whether the figures with any amount of yellow (red) on them are the same (more) in number as the figures with both yellow and red on them. The logical operation characterizing the psychological process is the composition (multiplication) of classes where each of the component members of one class is set in one-to-one correspondence with each of the component members of a second class such that A_1 (triangles with yellow on them) x A_2 (triangles with red on them) = A_1A_2 (triangles with both yellow and red on them). 4 trials.
2. In reference to the same stimuli, the S is asked whether there would be any figures with both yellow and red on them

remaining if all the circles with yellow (red) on them were removed. The logical operation characterizing the psychological process is the inversion (division) of a product class by one class such that A_1A_2 (triangles with both yellow and red on them) $\div$ A_1 (triangles with yellow on them) = A_2 (triangles with red on them). 4 trials. (The same procedure in 1 and 2 above is repeated with circles.)

Groupement IV: Co-univocal Multiplication of Classes (Score Range = 0 to 8)

1. In reference to triangles, some red, some yellow, and circles, all yellow, the S is asked whether the yellow circles are the same (more) in number as the yellow figures. The logical operation characterizing the psychological process is the composition (multiplication) of classes in which one member of one class is set in correspondence with several members of a second class such that K_1 [triangles (A_1 = red, B_1 = yellow)] x K_2 [yellow (A_2 = triangles, A_2' = circles)] = A_1A_2 (red triangles) + B_1A_2 (yellow triangles) + B_1A_2' (yellow circles). 4 trials.
2. In reference to the same stimuli, the S is asked whether there would be any yellow circles remaining if all the yellow figures were removed. The logical operation characterizing the psychological process is the inversion (division) of product classes by one class such that [A_1A_2 (red triangles) + B_1A_2 (yellow triangles) + B_1A_2' (yellow circles)] $\div$ [B_1 (yellow) + A_2 (triangles) + A_2' (circles)] = A_1 (red). 4 trials. (The same procedures in 1 and 2 above are repeated with a second stimulus set.

Groupement V: Addition of Asymmetrical Relations (Score Range = 0 to 8)

1. In reference to 3 sticks differing in length by a small amount, the S is shown the 3 sticks and told that stick A is shorter than stick B, that B is shorter than C, and is asked the existing length relationship between A and C. (The above procedure is repeated with the weight dimension.) The logical operation characterizing the psychological process is the composition (addition) of difference relations [(A $\leftarrow$ B) + (B $\leftarrow$ C) = (A $\leftarrow$ C)]. 4 trials.
2. In reference to the same stimuli, the S is shown the 3 sticks and told that C is longer than B, that B is longer than A, and is asked the existing length relationship between C and A. (The above procedure is repeated with the weight dimension.) The logical operation characterizing the psychological process is the reciprocity of difference relations [(A $\leftarrow$ B) + (B $\leftarrow$ C) = (A $\leftarrow$ C)] + [(C $\rightarrow$ B) + (B $\rightarrow$ A) = (C $\rightarrow$ A)] = (A = A). 4 trials.

Groupement VI: Addition of Symmetrical Relations (Score Range = 0 to 8)

1. In reference to 3 sticks each of a different color, but of the same length, the S is shown the 3 sticks and told that stick A equals stick B in length, that B equals C in length, and is asked the existing length relationship between A and C. (The above procedure is repeated with weight dimension.) The logical operation characterizing the psychological process is the composition (addition) of equivalence relations $(A \leftrightarrow B) + (B \leftrightarrow C) = (A \leftrightarrow C)$. 4 trials.
2. In reference to the same stimuli, the S is shown the 3 sticks and told that C equals B, that B equals A, and is asked the existing length relationship between C and A. (The above procedure is repeated with the weight dimension.) The logical operation characterizing the psychological process is the reciprocity of equivalence relations $[(A \leftrightarrow B) + (B \leftrightarrow C) = (A \leftrightarrow C)] + [(C \leftrightarrow B) + (B \leftrightarrow A) = (C \leftrightarrow A)] = (A \leftrightarrow A)$. 4 trials.

Groupement VII: Bi-univocal Multiplication of Relations (Score Range = 0 to 8)

1. In reference to 3 sticks differing in both length and weight, the S is shown the 3 sticks and told that A is both shorter and lighter than B, that B is both shorter and lighter than C, and is asked the existing length and weight relationships between A and C. The logical operation characterizing the psychological process is the composition (multiplication) of difference relations $(A \leftrightarrow B) \times (B \leftrightarrow C) = (A \leftrightarrow C)$. 4 trials.
2. In reference to the same stimuli, the S is shown the 3 sticks and told that C is both longer and heavier than B, that B is both longer and heavier than A, and is asked the existing length and weight relationships between C and A. The logical operation characterizing the psychological process is the reciprocity of difference relations $(A \leftrightarrow C) \div (A \leftrightarrow C) = (A \leftrightarrow A)$ which also may be expressed as $[(C \rightarrow B) \times (B \rightarrow A)] \div (C \rightarrow A) = A \leftrightarrow A$. 4 trials.

Groupement VIII: Co-univocal Multiplication of Relations (Score Range = 0 to 8)

1. In reference to 3 sticks differing in length, but of the same weight, the S is shown the 3 sticks and told that A is shorter and is the same weight as B, that B is shorter and is the same weight as C, and is asked the existing length and weight relationships between A and C. The logical operation characterizing the psychological process is the composition (multiplication) of difference relations by equivalence relations $(A \leftrightarrow B) \times (B \leftrightarrow C) = (A \leftrightarrow C)$. 4 trials.

2. In reference to the same stimuli, the S is shown the 3 sticks and told that C is longer and is the same weight as B, that B is longer and is the same weight as A, and is asked the existing length and weight relationships between C and A. The logical operation characterizing the psychological process is the reciprocity of difference relations by equivalence relations (A $\leftrightarrow\!\!\!\!+$ C) + (A $\leftrightarrow\!\!\!\!+$ C) = (A $\overset{\circ}{\leftrightarrow}\!\!\!\!+_{\circ}$ A) which may also be expressed as [(C $\leftrightarrow\!\!\!\!+$ B) x (B $\leftrightarrow\!\!\!\!+$ A)] + (C $\leftrightarrow\!\!\!\!+$ A) = (A $\overset{\circ}{\leftrightarrow}\!\!\!\!+_{\circ}$ A). 4 trials.

The combinatorial reasoning task asked the S to form all possible (28) pairs of 8 differently colored sets of chips without repeating pairs. The score range is 0 to 28.

For the some-all task, 2 items represent the relationship that all members of a subclass are members of a higher class. Two other items represent the relationship that some but not all members of a class are members of a given subclass.

Third Assessment Session Tasks

In the length transitivity task, the S compares (1) one stick A to another stick B in terms of their equal lengths, (2) stick A to a third stick C in terms of their unequal lengths, and judges the inequality and direction of difference of length of sticks B and C. The same procedure is carried out with clay balls in the weight transitivity task. There are 5 length and 5 weight case items.

In the identity conservation of length task the S (1) predicts the length of a single string to be the same if perceptually altered (3 judgments, 1 followed by an explanation inquiry), and (2) judges the length of the string to be the same after being perceptually altered (3 judgments, 1 followed by an explanation inquiry were required for the single alteration). Similar procedures are carried out with balls of clay in the weight identity case. In the equivalence of length task, the S (1) predicts the length of 2 perceptually equivalent strings to be equal if 1 was perceptually altered (3 judgments, 1 followed by an explanation inquiry), and (2) judges the length of the strings to be equal after 1 has been perceptually altered (3 judgments, 1 followed by an explanation inquiry). Similar procedures are carried out with balls of clay in the weight equivalence case. The score range for each of these conservation tasks is 0 to 6.

The class inclusion task includes 4 items in which the subject must decide whether there are more members of a superordinate class than members of a nested subclass, and 1 item assessing which of 2 intersecting classes is larger. The score range is 0 to 5.

III

RESULTS

PRELIMINARY CONSIDERATIONS

The subjects' performances for the initial assessment year were used to compute Hoyt reliability coefficients, formally analogous to coefficient alpha, and to obtain associated standard errors of measurement for all the concept tasks except dichotomous sorting and combinatorial reasoning.[1] In addition, inter-item biserial correlation coefficients were computed for the various combinations of the *groupements* test items (see Hooper, Brainerd, & Sipple, 1975, p. 21 and pp. 79-92). The Hoyt reliability values for the various *groupements* item subset comparisons (see Table 4) ranged from .62 to .88. In the cases of comparisons involving 16 or more items the values all exceeded .71. The corresponding values for the conservation tasks (see Table 5) ranged from .85 to .95 with an average value of .91. The Hoyt values for the seriation task series, transitivity of length, and transitivity of weight cases were .63, .94, and .91, respectively (see Table 6). For the some-all understanding, cardinality, and class inclusion tasks, the corresponding Hoyt values were .37, .72, and .67 (see Tables 7 and 8). Thus it may be concluded that, with the exception of the some-all understanding task, the reliability values indicate a generally acceptable level of internal consistency for the overall Piagetian concept task series.

Order of presentation analyses of the initial year's data included two comparisons. In the first case, the four presentation orders for the tasks administered in the first session (kindergarten and third grade subjects only, see page 9) were compared. Variance analyses indicated no significant presentation order effects on the performance of any measure. In the second case, the six presentation orders employed in the conservation and transitive inference task array were compared. With the exception of two cases, there was an absence of presentation order effects. At the preschool level, those subjects who were administered the length and weight transitivity tasks first (Orders 5 and 6) and last (Orders 1 and 4) in

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[1] These analyses were computed using the LERTAP test analysis program, Version 2, 1974, Madison Academic Computing Center, University of Wisconsin, Madison, Wisconsin.

TABLE 4

HOYT RELIABILITIES FOR VARIOUS *GROUPEMENT* SUBSCALES
COMBINING KINDERGARTEN, THIRD GRADE, AND SIXTH GRADE (N = 180)
(INITIAL ASSESSMENT YEAR'S RESULTS)

Subscale	Hoyt	Standard Error of Measurement
All (64) *Groupement* Items	.88	3.01
All (32) Class *Groupement* (I-IV) Items	.85	2.21
All (32) Relational *Groupement* (V-VIII) Items	.84	1.90
All (32) Composition *Groupement* Items	.81	2.32
All (32) Inversion/Reciprocity *Groupement* Items	.84	1.81
All (16) Class Composition *Groupement* Items	.85	1.61
All (16) Class Inversion *Groupement* Items	.78	1.29
All (16) Relational Composition *Groupement* Items	.71	1.44
All (16) Relational Reciprocity *Groupement* Items	.75	1.18
All (8) *Groupement* I Items	.62	.91
All (8) *Groupement* II Items	.67	.92
All (8) *Groupement* III Items	.69	1.13
All (8) *Groupement* IV Items	.76	1.03
All (8) *Groupement* V Items	.67	.81
All (8) *Groupement* VI Items	.78	.62
All (8) *Groupement* VII Items	.75	.93
All (8) *Groupement* VIII Items	.65	.99

TABLE 5

HOYT RELIABILITIES FOR VARIOUS CONSERVATION SUBSCALES
COMBINING KINDERGARTEN AND THIRD GRADE (N = 120)
(INITIAL ASSESSMENT YEAR'S RESULTS)

Subscale	Hoyt	Standard Error of Measurement
All (7) Conservation of Number Items	.87	.72
All (24) Conservation of Length and Weight Items (Supporting Explanation Required)	.95	1.75
All (24) Conservation of Length and Weight Items (Supporting Explanation Not Required)	.94	1.72
All (12) Conservation of Length Items (Supporting Explanation Required)	.94	1.06
All (12) Conservation of Length Items (Supporting Explanation Not Required)	.94	1.03
All (12) Conservation of Weight Items (Supporting Explanation Required)	.93	1.13
All (12) Conservation of Weight Items (Supporting Explanation Not Required)	.93	1.09
All (12) Identity Conservation Items (Supporting Explanation Required)	.87	1.24
All (12) Identity Conservation Items (Supporting Explanation Not Required)	.85	1.17
All (12) Equivalence Conservation Items (Supporting Explanation Required)	.93	1.12
All (12) Equivalence Conservation Items (Supporting Explanation Not Required)	.93	1.14
All (12) Conservation Prediction Items (Supporting Explanation Required)	.88	1.26
All (12) Conservation Prediction Items (Supporting Explanation Not Required)	.87	1.23
All (12) Conservation Judgment Items (Supporting Explanation Required)	.91	1.17
All (12) Conservation Judgment Items (Supporting Explanation Not Required)	.91	1.15

TABLE 6

HOYT RELIABILITIES FOR THE SERIATION, TRANSITIVITY OF LENGTH, AND TRANSITIVITY OF WEIGHT SUBSCALES COMBINING KINDERGARTEN AND THIRD GRADE (N = 120) (INITIAL ASSESSMENT YEAR'S RESULTS)

Subscale	Hoyt	Standard Error of Measurement
All (5) Seriation Items	.63	.71
All (5) Transitivity of Length Items	.94	.39
All (5) Transitivity of Weight Items	.91	.33
All (10) Transitivity of Length and Weight Items Combined	.87	.81

TABLE 7

HOYT RELIABILITIES FOR THE SOME-ALL AND CARDINALITY TASKS COMBINING KINDERGARTEN AND THIRD GRADE (N = 120) (INITIAL ASSESSMENT YEAR'S RESULTS)

Subscale	Hoyt	Standard Error of Measurement
All (4) Some-All Items	.37	.45
All (5) Cardinality Items	.72	.72

TABLE 8

HOYT RELIABILITIES FOR SUBSCALES OF THE CLASS INCLUSION TASK COMBINING KINDERGARTEN, THIRD, AND SIXTH GRADES (INITIAL ASSESSMENT YEAR'S RESULTS)

Subscale	Hoyt	Standard Error of Measurement
Class Inclusion Items 1 through 4	.63	.63
All (5) Class Inclusion Items	.67	.75

the series significantly outperformed those subjects who received these tasks second (Orders 2 and 3). Differences were significant for the length ($\chi^2 = 9.62$, df = 2, $p < .01$) and weight cases ($\chi^2 = 8.75$, df = 2, $p < .02$).[2]

Finally a comparison of school location effects for the initial year's *groupements* performance patterns was conducted (Dihoff, 1975). Analyses of variance indicated a notable absence of significant main effects or higher order interactions (sex and grade-level factors were included in these analyses, see Dihoff, 1975, pp. 31-32). Only in the cases of *groupement* I ($F = 5.31$, df = 3, 156, $p < .002$) and *groupement* II ($F = 7.54$, df = 3, 156, $p < .001$) were the school location effects significant. Inspection of the individual school means indicated that the performances of the students of one school were significantly inferior to those of the other three schools. In regard to this difference, it may be noted that the socioeconomic status of the parents of the children attending this school was approximately equal to that of one of the other three schools, but lower than two schools. In addition, the proportion of minority group students was higher than that of two of the schools, and lower than the remaining school. In view of these minimal distinctions, school conditions were combined for all subsequent analyses in the present investigation.

PRIMARY ANALYSES

Normative Performance Patterns--Initial Year Data

The means and standard deviations by grade level and sex for the various concept task arrays and memory measures for the total subject sample (N = 180) are presented in Tables 9 to 11. While pass/fail and stage level criteria were employed in a number of previous reports, i.e., Dihoff, 1975; Gonchar, 1975; Hooper and Toniolo, 1976; Toniolo and Hooper, 1975, the present analyses will be concerned exclusively with interval data. Factorial variance analyses revealed significant grade level effects for all the Piagetian concept tasks with the exception of length transitivity. Grade-level main effects were significant for all three indices of memoric ability. The only instance of a significant sex difference was observed for the class inclusion task where females outperformed males at all age-grade levels. Significant grade level/sex interactions were notably absent for all indices except length conservation (equivalence task format), $F = 4.61$, $df = 2, 116$, $p < .05$, where kindergarten female subject superiority and third grade male subject superiority was observed. It may be concluded, therefore, that children's performance on the present

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[2] In the initial assessment year, the preschool subsample was not administered any measures except the conservation and transitivity tasks, and this subsample was not included for longitudinal assessment in the second year.

TABLE 9

MEANS AND STANDARD DEVIATIONS FOR THE FOUR CLASS *GROUPEMENT* TASKS FROM THE INITIAL ASSESSMENT YEAR'S RESULTS

Grade	*Groupements* I $\bar{X}$	I S.D.	II $\bar{X}$	II S.D.	III $\bar{X}$	III S.D.	IV $\bar{X}$	IV S.D.	Total Class $\bar{X}$	Total Class S.D.
Kindergarten										
Males (n = 30)	4.13	1.61	4.63	1.63	3.57	1.38	3.93	1.68	16.27	3.95
Females (n = 30)	4.63	1.38	5.00	2.00	3.23	1.72	3.80	1.99	16.67	5.03
Total (n = 60)	4.38	1.51	4.82	1.82	3.40	1.55	3.87	1.83	16.47	4.49
Third										
Males (n = 30)	4.53	1.25	4.80	1.35	3.13	2.10	4.57	2.29	17.03	4.92
Females (n = 30)	4.53	1.25	4.63	1.16	3.70	2.59	5.23	2.13	18.10	4.66
Total (n = 60)	4.53	1.24	4.72	1.25	3.42	2.35	4.90	2.22	17.57	4.78
Sixth										
Males (n = 30)	5.93	1.39	6.10	1.49	3.97	2.14	6.07	1.96	22.07	5.36
Females (n = 30)	6.37	1.30	6.67	1.35	4.97	2.47	6.40	2.19	24.40	5.46
Total (n = 60)	6.15	1.35	6.38	1.44	4.47	2.35	6.23	2.07	23.23	5.49
Combined Grade										
Males (n = 90)	4.87	1.61	5.18	1.62	3.56	1.91	4.86	2.17	18.46	5.38
Females (n = 90)	5.18	1.55	5.43	1.77	3.97	2.38	5.14	2.34	19.72	6.04
Total (N = 180)	5.02	1.58	5.31	1.70	3.76	2.17	5.00	2.25	19.09	5.74

TABLE 10

MEANS AND STANDARD DEVIATIONS FOR THE FOUR RELATIONS
GROUPEMENT TASKS FROM THE INITIAL ASSESSMENT YEAR'S RESULTS

	Groupements									
	V		VI		VII		VIII		Total Relations	
Grade	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Kindergarten										
Males (n = 30)	6.20	1.63	6.73	1.70	5.50	2.10	5.23	1.78	23.67	5.19
Females (n = 30)	6.27	1.53	6.20	1.85	5.43	2.08	4.90	1.81	22.80	5.14
Total (n = 60)	6.23	1.57	6.47	1.78	5.47	2.07	5.07	1.78	23.23	5.14
Third										
Males (n = 30)	6.83	1.46	7.83	.53	6.43	2.11	6.43	1.43	27.53	3.96
Females (n = 30)	7.17	1.32	7.70	1.02	6.10	2.34	6.07	1.89	27.03	3.87
Total (n = 60)	7.00	1.39	7.77	.81	6.27	2.22	6.25	1.67	27.28	3.89
Sixth										
Males (n = 30)	7.70	2.22	7.60	1.13	7.00	1.20	7.17	1.21	29.47	2.70
Females (n = 30)	7.07	1.72	7.63	1.16	6.83	1.44	6.90	1.40	28.43	3.87
Total (n = 60)	7.38	1.34	7.62	1.14	6.92	1.32	7.03	1.30	28.95	3.35
Combined Grade										
Males (n = 90)	6.91	1.45	7.39	1.30	6.31	1.94	6.28	1.68	26.89	4.71
Females (n = 90)	6.83	1.57	7.18	1.54	6.12	2.05	5.96	1.88	26.09	4.92
Total (N = 180)	6.87	1.51	7.28	1.42	6.22	1.99	6.12	1.79	26.49	4.82

TABLE 11

MEANS AND STANDARD DEVIATIONS FOR CLASS INCLUSION, COMBINATORIAL REASONING, AUDITORY MEMORY SPAN TEST, AUDITORY SEQUENTIAL MEMORY TEST, AND VISUAL ORIENTATION MEMORY FROM THE INITIAL ASSESSMENT YEAR'S RESULTS

Grade	CI		CR		AMS		ASM		VOM	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Kindergarten										
Males (n = 30)	2.13	1.48	7.00	5.62	23.80	5.56	20.93	9.19	9.07	3.27
Females (n = 30)	2.87	1.41	8.00	6.92	25.60	10.63	23.67	13.67	9.00	3.11
Total (n = 60)	2.50	1.48	7.50	6.27	24.70	8.46	22.30	11.63	9.03	3.16
Third										
Males (n = 30)	3.57	1.11	18.97	8.18	30.93	8.03	30.13	10.35	13.57	2.85
Females (n = 30)	3.87	1.20	19.83	8.73	30.43	9.93	27.17	10.58	13.57	3.65
Total (n = 60)	3.72	1.15	19.40	8.40	30.68	8.96	28.65	10.48	13.57	3.24
Sixth										
Males (n = 30)	4.17	1.12	24.17	6.15	34.27	9.49	34.10	10.07	17.13	1.68
Females (n = 30)	4.37	1.07	25.07	6.42	36.37	10.05	36.00	12.29	17.77	1.99
Total (n = 60)	4.27	1.09	24.17	6.25	35.32	9.75	35.05	11.18	17.45	1.85
Combined Grade										
Males (n = 90)	3.29	1.50	16.71	9.83	26.67	8.93	28.39	11.23	13.26	4.25
Females (n = 90)	3.70	1.37	17.63	10.27	30.80	11.02	28.94	13.19	13.44	4.66
Total (N = 180)	3.49	1.45	17.17	10.04	30.23	10.02	28.67	12.22	13.35	4.45

tasks is significantly related to chronological age and generally free of notable sex biases.

In terms of *groupements* relative item difficulty, the means presented in Tables 9 and 10 indicate the following rank order of increasing difficulty: *Groupement* VI < V < VII < VIII < II < IV < I < III. This pattern is essentially borne out in pass/fail and scaling analyses (Dihoff, 1975). In general agreement with Brainerd's (1972) results, the composite means of the initial year's data (see Tables 9 and 10) for the relational *groupements*, 26.49, and classificatory *groupements*, 19.09, clearly indicate the former tasks to be of lesser overall difficulty. Insofar as the composition versus reverse operations are concerned, little difference was observed for the four relational cases. In contrast, the class composition operation was clearly more difficult than its inverse counterpart (see Dihoff, 1975, p. 22) and this was true for all four cases.

For the additional classificatory measures (see Table 12) certain definitive performance patterns are evident. Some-all understanding is obviously a much easier task when compared to class inclusion. This is in accord with previous investigations (e.g., Hooper, Sipple, Goldman, & Swinton, 1974; Kofsky, 1966). The majority of the kindergarten children produced a single consistent and exhaustive dichotomous sorting while two dichotomies were produced by the majority of the third graders (see also, Hooper, Sipple, Goldman, & Swinton, 1974, p. 16). As anticipated, the combinatorial reasoning task was the most difficult classificatory measure (see Table 11).

A separate analysis of the conservation and transitive inference tasks (see Tables 13 and 14) has been completed (Brainerd & Hooper, 1975; Toniolo & Hooper, 1975). As mentioned above this analysis included an additional sample of 60 younger preschool children. In summary:

> The design was a 3/2/2/2 mixed-model analysis of variance. The factors were age (preschool/kindergarten/ third grade), task (identity/equivalence), criterion (judgment only/judgment plus explanation) and content area (length/weight).
>
> The main effects of age, task, and criterion were large and highly significant. Equivalence conservation was observed to be of greater difficulty than identity conservation. More trials were passed under the judgment only criterion than with a judgment plus explanation criterion. Significant interactions of Age x Task, Task x Criterion, and Age x Criterion were also observed. Equivalence tasks were found to be more difficult than identity tasks for preschoolers and kindergarteners but not for third graders. Performance differences between the identity and equivalence tasks were greater with a judgment only criterion than with a judgment plus explanation criterion, and these

TABLE 12

MEANS AND STANDARD DEVIATIONS FOR THE SOME-ALL, DICHOTOMOUS SORTING, CARDINALITY, AND ARITHMETIC ADDITION TASKS FROM THE INITIAL ASSESSMENT YEAR'S RESULTS

Grade	Some-All		Dichotomous Sorting		Cardinality		Arithmetic Addition	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Kindergarten								
Males (n = 30)	3.43	.73	1.13	.82	1.90	1.27	5.13	5.20
Females (n = 30)	3.57	.82	1.07	.87	1.67	1.21	5.40	5.08
Total (n = 60)	3.50	.77	1.10	.84	1.78	1.24	5.27	5.10
Third								
Males (n = 30)	3.80	.48	2.10	.76	3.93	1.05	15.57	.90
Females (n = 30)	3.87	.43	2.10	.80	3.60	1.07	15.57	.90
Total (n = 60)	3.83	.46	2.10	.78	3.77	1.06	15.57	.89
Combined Grade								
Males (n = 60)	3.62	.64	1.62	.92	2.92	1.54	10.35	6.43
Females (n = 60)	3.72	.67	1.58	.98	2.63	1.49	10.48	6.28
Total (N = 120)	3.67	.65	1.60	.95	2.78	1.52	10.42	6.33

TABLE 13

MEANS AND STANDARD DEVIATIONS FOR THE NUMBER CONSERVATION, LENGTH CONSERVATION, AND WEIGHT CONSERVATION TASKS FROM THE INITIAL ASSESSMENT YEAR'S RESULTS

Grade	Number Conservation		Length Conservation				Weight Conservation			
			Identity		Equivalence		Identity		Equivalence	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Kindergarten										
Males (n = 30)	4.07	2.26	3.13	1.80	1.93	2.12	3.57	1.76	2.13	2.42
Females (n = 30)	4.27	2.35	3.93	1.70	2.63	2.31	3.63	1.99	2.47	2.30
Total (n = 60)	4.17	2.29	3.53	1.78	2.28	2.23	3.60	1.86	2.30	2.35
Third										
Males (n = 30)	6.73	.83	5.23	1.46	5.13	1.96	5.50	1.36	5.10	2.11
Females (n = 30)	6.60	1.04	4.83	1.72	4.03	2.72	4.33	2.06	4.00	2.72
Total (n = 60)	6.67	.93	5.03	1.59	4.58	2.42	4.92	1.83	4.55	2.47
Combined Grade										
Males (n = 60)	5.40	2.16	4.18	1.94	3.53	2.59	4.53	1.84	3.62	2.70
Females (n = 60)	5.43	2.15	4.38	1.76	3.33	2.60	3.98	2.04	3.23	2.61
Total (N = 120)	5.42	2.14	4.28	1.84	3.43	2.59	4.26	1.95	3.43	2.65

TABLE 14

MEANS AND STANDARD DEVIATIONS FOR THE SERIATION, LENGTH TRANSITIVITY, AND WEIGHT TRANSITIVITY TASKS FROM THE INITIAL ASSESSMENT YEAR'S RESULTS

Grade	Seriation		Length Transitivity		Weight Transitivity	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Kindergarten						
Males (n = 30)	1.87	1.17	3.60	1.99	4.33	1.27
Females (n = 30)	2.00	.98	3.67	1.99	4.27	1.60
Total (n = 60)	1.93	1.07	3.63	1.97	4.30	1.43
Third						
Males (n = 30)	3.80	.89	4.13	1.89	4.67	1.27
Females (n = 30)	3.67	.66	4.43	1.36	4.93	.37
Total (n = 60)	3.73	.78	4.28	1.64	4.80	.94
Combined Grade						
Males (n = 60)	2.83	1.42	3.87	1.94	4.50	1.27
Females (n = 60)	2.83	1.18	4.05	1.73	4.60	1.20
Total (N = 120)	2.83	1.30	3.96	1.84	4.55	1.23

> differences between the two criteria were more pronounced with preschoolers and kindergarteners than with third graders. The preschool and kindergarten subsamples did not differ. Comparisons of the relative difficulty of the identity versus equivalence conservation cases, utilizing a dichotomous pass/fail scoring criterion, were considerably less persuasive, i.e., only the kindergarten and total sample weight cases indicated a significant lesser difficulty for identity-conservation. A significant performance improvement at every grade level for the transitivity tasks was observed, with the largest differences between the preschool and kindergarten subsamples. The transitivity tasks were significantly easier than all conservation measures at the preschool and kindergarten level, but at the third grade level, only the transitivity of weight/conservation of weight comparison was significant [Toniolo & Hooper, 1975, p. ix].

The measures in the present concept task array which may be conceptually and developmentally related to the emergent understanding of a natural number concept have also been discussed previously (Gonchar, 1975). These comparisons included the class indices of dichotomous sorting (see Table 12) and class inclusion (see Table 11), the relational measures of seriation and serial correspondence (see Table 14), and the number concept tasks dealing with cardinality (see Table 12) and conservation (see Table 13). Subsidiary analyses dealt with Brainerd's (1973a, 1973b) transitive inference, cardinality, and arithmetic operations measures which were also drawn from the present task battery. Gonchar (1975) summarized his primary results as follows:

> The present two-phase study sought to investigate the structural isomorphic relations between the cognitive skills which presumably underlie number competence as formalized by Piaget (1966) and the actual performance patterns of children when confronted with task problems requiring logical reasoning and involving number-related concepts for their solutions. In the main study, the accuracy of Piaget's predictions concerning the developmental priority and inter-patterning of relations between emergent cognitive competencies of classes and relations was assessed. Since his theory contends that number is in co-development with and a resultant of the coordinated synthesis of classes and relations, measures of the developmental relatedness and concurrence of conceptual performance patterns were applied in assessing the appropriateness of Piaget's model. In the second phase of this investigation, an attempt was made to reconcile Piaget's model and the recent experimental evidence presented by Brainerd (1972, 1973a, 1973b, 1973c, 1974a) in support of an ordinal sequence

in the development of the natural number concept.

Initial discussion focused on the study's attempt to corroborate the philosophical and observational techniques of inquiry in psychological investigations by (a) presenting by detailed theoretical explication the differing interpretations of natural number as viewed in the philosophies of intuitionism, logicism, and formalism and (b) describing the psychological role of traditional logic in identifying and characterizing the elements in developmental processes.

In the main study, two assessment situations for each concept were adapted (including the measures employed by Piaget & Szeminska, 1952; Kofsky, 1963; and Brainerd, 1973a) as those being the behavioral isomorphs of the formal definitions of natural number derived from mathematical philosophy and its partitioned elements of classes and relations. Tasks within a concept area were weighted on their relative importance to the acquisition of the particular concept and to the differential degree of difficulty reflected in certain specified task features. In the second analysis, a set of behavioral indices similar to those chosen by Brainerd as appropriate in the study of natural number concept was employed. In both studies, behavioral indices of competence were specified and included within a total scoring scheme employing a standard criterion across concepts and a three-stage qualitative model of conceptual acquisition analogous to that developed by Piaget. The entire task battery was administered to a core group of 60 kindergarteners and 60 third graders as part of a 4-year, longitudinal investigation with annual assessments.

The results of the main study suggested a developmental picture in which the concepts of classes and relations evolve co-jointly with the natural number concept among which a universal and invariant sequence is not apparent. While 53% of the subject sample demonstrated synchronous development (concordant rankings on all three concepts) in reference to the number-related concepts, the remaining subjects demonstrated a number of alternative and potentially ambiguous sequential patterns. The relevant data consisting of triads of cognitive developmental acquisitions did not reveal a high degree of sequential regularity across same age samples or an invariant pattern between age levels. The natural number concept seems to develop through an extended process of concordant and discordant patterning. While some children acquired the set of cognitive elements in one consistent (co-emerging) pattern, others acquired it in a number of other patterns. This does not necessarily suggest the absence of any functional developmental connection among the elements of relations, classes, and number. The strong values of association registered between concepts, the high proportion of subjects with concordant rankings, and the lack of a clear developmental sequence suggested that there is a possible underlying affiliation between

the concept of relations and the concept of classes in the construction of the natural number concept. In contrast, experiments using the tasks suggested by Brainerd result in confirming evidence for the ordinal theory of natural number development. It was argued that these effects reflect the differential difficulty of Brainerd's concept tasks and do not accurately assess developmental acquisitions.

In sum, the findings of this investigation are in agreement with Piaget's contention that cognitive development is dialectical in nature. Without further longitudinal follow-up, it would be conjecture to conclude definitively from the data that Piaget's model of number development (being the resultant of the synchronous emergence of relational and classification competencies) is upheld [Gonchar, 1975, pp. ix-x].

Normative Performance Patterns--Second Year Data

Two initial issues of concern are the questions dealing with survival-participation effects and repeated measurement factors.[3] With regard to the former issue, moderate sample attrition (13.3%) occurred for the longitudinal core group at the time of second testing (see Table 3). Twelve subjects (four males and eight females) at the kindergarten level, six subjects (four males and two females) at the third grade level, and six subjects (four males and two females) did not participate one year later as first, fourth, and seventh grades in the second assessment year. Comparison of this group's (N = 24) first year performances with the survivor's (N = 156) initial year scores indicated nonparticipant superiority on the class *groupement* II task (means of 5.50 versus 4.69), $t = 2.40$, $df = 178$, $p < .01$; and similar differences on relational *groupement* VII (means of 6.75 versus 5.76) $t = 2.07$, $df = 178$, $p < .025$. For the younger subject subsample three comparisons proved to be significant. Continuing participant's (N = 102) some-all understanding task scores were superior to the "dropout" group's (N = 18) performances (i.e., means of 3.74 versus 3.27), $t = 2.79$, $df = 118$, $p < .005$. In similar fashion, the noncontinuing group was inferior to the continuing longitudinal core sample on the arithmetic addition task (means of 7.50 versus 10.93, $t = 2.14$, $df = 118$, $p < .025$) and cardinality task (means of 2.17 versus 2.88, $t = 1.85$, $df = 118$, $p < .05$). All other

---

[3] Year two data were also examined to determine presentation order effects on the tasks in session one. The only measure where performance was significantly effected was the seriation task occuring at the fourth grade level only. Performance was significantly higher for orders one and three. It is noted that within order one the seriation test followed the dichotomies and cardinality tasks, and within order three the seriation task was presented last in the group of five tasks. In addition, the seriation task was first in the series within order two, and followed the arithmetic addition and conservation of number measures within order four.

comparisons were nonsignificant. It is clear, therefore, that few distinct performance biases are evident in the present longitudinal core sample and that the continuing subjects may be considered representative of the original age-level cohorts.

The second question of repeated measurement effects evaluation was examined by comparisons of the longitudinal sample's second year performances to those demonstrated by a matched cohort control group assessed for the first time in the spring of 1974 (see Tables 15 to 20). Factorial variance analyses (sex, grade level, and longitudinal group versus control group) were employed. For the subset of measures which were administered to all subjects the evaluations indicated significant age-grade level main effects for 14 of 15 indices (see analysis summaries presented in Tables 21 and 22). Group differences (repeated measurement effects) approached significance for the class inclusion task ($F = 3.59$, $df = 1, 216$, $.10 < p > .05$) and for the combined class *groupements* (I-IV) score ($F = 3.89$, $df = 1, 216$, $p = .05$). In both cases the core longitudinal group scores were consistently higher at all age-grade levels. Sex differences and higher order interactions were generally nonsignificant for all the variables.

In the case of the first and fourth grade subjects, who received the full task battery, no cases of significant testing main effects were observed (see summaries presented in Tables 23 to 25). However, two instances of significant treatment group/grade level interactions resulted, i.e., conservation of length (identity task format) $F = 6.48$ $df = 1, 142$, $p < .05$, and conservation of length (equivalence task format) $F = 5.63$ $df = 1, 142$, $p < .05$. Inspection of the respective group means at each grade level indicated core longitudinal sample superiority (means of 4.46 versus 3.63; $t = 2.88$, $df = 70$, $p < .01$) at the first grade level and control group superiority (means of 5.83 versus 5.30; $t = 2.12$, $df = 76$, $p < .05$) at the fourth grade for the length conservation identity task.

Similarly, in the equivalence format length conservation task case, the first grade longitudinal sample subjects were superior to their control group counterparts (means of 4.04 versus 2.50; $t = 4.83$, $df = 70$, $p < .001$) while the converse relationship was true for the fourth grade comparisons (means of 4.96 versus 5.33; $t = 1.25$, $df = 76$, N.S.).

In terms of the overall logical concept task array, it may be concluded that there is minimal evidence for pervasive repeated measurement effects in the subsequent year's assessments.

The second year performances contrasted with the initial year's assessment results of the present longitudinal sample may be briefly summarized (see Tables 26 to 31). As in the first year analyses, sex differences were notably absent. Mixed model analyses of variance (sex, grade level, year of measurement) indicated the following general patterns. Grade level main effects were significant for all measures (in the case of weight transitivity the F value approached significance, $p < .06$, see Tables 32 to 36). Sex difference main effects were consistently nonsignificant except for the class inclusion tasks where females outperformed males ($F = 4.70$, $df = 1, 150$, $p < .03$). Sex/grade level interactions were nonsignificant except for the weight conservation (equivalence format) task ($F = 5.10$, $df = 1, 98$, $p < .03$).

TABLE 15

MEANS AND STANDARD DEVIATIONS FOR THE FOUR CLASS GROUPEMENT TASKS ADMINISTERED TO THE LONGITUDINAL AND TESTING CONTROL SUBJECTS

Assessment Condition and Grade	Groupements								Total Cl
	I		II		III		IV		
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$
Longitudinal Sample									
First									
Males (n = 26)	4.73	.92	5.31	1.59	3.81	2.37	4.85	2.51	18.69
Females (n = 22)	4.82	1.22	5.05	1.46	3.64	2.22	4.09	2.20	17.59
Total (n = 48)	4.77	1.06	5.19	1.53	3.73	2.28	4.50	2.38	18.19
Fourth									
Males (n = 26)	5.81	1.74	5.65	1.60	4.27	2.16	6.50	1.77	22.23
Females (n = 28)	5.57	1.69	5.64	1.57	4.25	2.22	5.82	2.45	21.25
Total (n = 54)	5.69	1.70	5.63	1.57	4.26	2.17	6.15	2.16	21.72
Seventh									
Males (n = 26)	6.23	1.56	6.50	1.50	4.31	2.07	6.73	2.13	23.77
Females (n = 28)	6.32	1.66	6.57	1.64	5.68	2.16	6.89	1.91	25.46
Total (n = 54)	6.28	1.60	6.54	1.56	5.02	2.21	6.82	2.00	24.65
Combined Grade									
Males (n = 78)	5.59	1.57	5.82	1.63	4.13	2.19	6.03	2.29	21.56
Females (n = 78)	5.63	1.65	5.80	1.67	4.59	2.33	5.72	2.44	21.73
Total (N = 156)	5.61	1.60	5.81	1.64	4.36	2.27	5.87	2.37	21.65
Testing Control Sample									
First									
Males (n = 12)	4.67	.78	5.42	.90	3.75	2.05	5.25	2.18	19.08
Females (n = 12)	4.83	1.53	4.42	1.62	3.50	2.19	3.75	2.14	16.50
Total (n = 24)	4.75	1.19	4.92	1.38	3.63	2.08	4.50	2.25	17.79
Fourth									
Males (n = 12)	4.83	1.47	4.92	1.56	3.92	2.61	5.50	2.84	19.17
Females (n = 12)	5.33	1.44	5.25	1.36	3.83	1.80	5.33	2.67	19.75
Total (n = 24)	5.08	1.44	5.08	1.44	3.88	2.19	5.42	2.70	19.46
Seventh									
Males (n = 12)	6.00	1.86	6.17	1.85	3.75	2.26	5.00	2.80	20.92
Females (n = 12)	6.25	1.82	6.33	1.72	4.67	2.53	6.50	1.98	23.75
Total (n = 24)	6.13	1.80	6.25	1.75	4.21	2.40	5.75	2.49	22.33
Combined Grade									
Males (n = 36)	5.17	1.52	5.50	1.54	3.81	2.25	5.25	2.56	19.72
Females (n = 36)	5.47	1.66	5.33	1.72	4.00	2.19	5.19	2.49	20.00
Total (N = 72)	5.32	1.59	5.42	1.63	3.90	2.21	5.22	2.51	19.86

TABLE 16

MEANS AND STANDARD DEVIATIONS FOR THE FOUR RELATIONS
ARRANGEMENT TASKS ADMINISTERED TO THE LONGITUDINAL AND TESTING CONTROL SUBJECTS

Assessment Condition and Grade	Groupements								Total Relati	
	V		VI		VII		VIII			
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S
Longitudinal Sample										
First										
Males (n = 26)	7.50	.86	7.35	1.79	6.50	1.96	6.35	1.85	27.69	4.
Females (n = 22)	6.82	1.74	6.77	1.72	6.18	2.02	5.86	1.73	25.64	4.
Total (n = 48)	7.19	1.36	7.08	1.76	6.35	1.97	6.13	1.79	26.75	4.
Fourth										
Males (n = 26)	7.39	1.13	7.89	.33	7.19	1.30	6.89	1.28	29.35	2.
Females (n = 28)	7.18	1.28	7.43	1.26	7.18	1.19	6.75	1.46	28.54	2.
Total (n = 54)	7.28	1.20	7.65	.95	7.19	1.23	6.82	1.36	28.93	2.
Seventh										
Males (n = 26)	7.54	.99	7.89	.43	7.08	1.92	7.58	.81	30.08	3.
Females (n = 28)	7.71	.66	7.93	.38	7.18	1.44	7.21	1.20	30.04	2.
Total (n = 54)	7.63	.83	7.91	.40	7.13	1.67	7.39	1.04	30.06	2.
Combined Grade										
Males (n = 78)	7.47	.99	7.71	1.09	6.92	1.76	6.94	1.45	29.04	3.
Females (n = 78)	7.27	1.30	7.42	1.27	6.90	1.59	6.67	1.53	28.26	3.
Total (N = 156)	7.37	1.15	7.56	1.19	6.91	1.67	6.80	1.50	28.65	3.
Testing Control Sample										
First										
Males (n = 12)	7.00	1.41	6.58	2.11	6.50	1.45	5.67	1.72	25.75	3.
Females (n = 12)	7.08	1.38	7.08	1.56	6.67	1.15	6.17	1.75	27.00	4.
Total (n = 24)	7.04	1.37	6.83	1.83	6.58	1.28	5.92	1.72	26.37	3.
Fourth										
Males (n = 12)	7.83	.39	7.75	.62	7.33	1.23	7.08	1.38	30.00	2.
Females (n = 12)	7.50	1.00	7.83	.58	7.75	.62	7.08	1.08	30.17	1.
Total (n = 24)	7.67	.76	7.79	.59	7.54	.98	7.08	1.21	30.08	2.
Seventh										
Males (n = 12)	7.75	.62	7.67	1.15	7.58	.79	7.25	.97	30.25	1.
Females (n = 12)	7.42	.90	7.92	.29	6.83	1.59	7.58	.67	29.75	2.
Total (n = 24)	7.58	.78	7.79	.83	7.21	1.28	7.42	.83	30.00	2.
Combined Grade										
Males (n = 36)	7.53	.97	7.33	1.49	7.14	1.25	6.67	1.53	28.67	3.
Females (n = 36)	7.33	1.10	7.61	1.02	7.08	1.25	6.94	1.35	28.97	3.
Total (N = 72)	7.43	1.03	7.47	1.28	7.11	1.24	6.81	1.44	28.82	3.

TABLE 17

MEANS AND STANDARD DEVIATIONS FOR CLASS INCLUSION, COMBINATORIAL REASONING, AUDITORY MEMORY SPAN TEST, AUDITORY SEQUENTIAL MEMORY TEST, AND VISUAL ORIENTATION MEMORY FOR THE LONGITUDINAL AND TESTING CONTROL SUBJECTS

Assessment Condition and Grade	CI $\bar{X}$	CI S.D.	CR $\bar{X}$	CR S.D.	AMS $\bar{X}$	AMS S.D.	ASM $\bar{X}$	ASM S.D.	VOM $\bar{X}$
Longitudinal Sample									
First									
Males (n = 26)	3.31	1.41	17.04	9.25	28.38	8.66	27.88	12.84	12.27
Females (n = 22)	3.36	1.18	16.55	8.90	34.95	12.03	32.68	16.78	12.41
Total (n = 48)	3.33	1.29	16.81	9.00	31.40	10.75	30.08	14.81	12.33
Fourth									
Males (n = 26)	4.08	1.13	23.50	6.13	37.54	10.76	36.23	16.20	17.00
Females (n = 28)	4.14	1.08	22.86	7.32	33.50	11.20	35.68	15.72	15.00
Total (n = 54)	4.11	1.09	23.17	6.72	35.44	11.08	35.94	15.80	15.96
Seventh									
Males (n = 26)	4.35	1.32	23.85	6.21	40.81	8.61	36.46	12.86	17.35
Females (n = 28)	4.61	.79	25.39	5.36	41.61	7.96	36.36	10.15	18.11
Total (n = 54)	4.48	1.08	24.65	5.78	41.22	8.21	36.41	11.42	17.74
Combined Grade									
Males (n = 78)	3.91	1.35	21.46	7.90	35.58	10.68	33.53	14.44	15.54
Females (n = 78)	4.09	1.12	21.99	7.96	36.82	10.91	35.08	14.21	15.38
Total (N = 156)	4.00	1.24	21.72	7.91	36.20	10.78	34.30	14.30	15.46
Testing Control Sample									
First									
Males (n = 12)	3.17	2.29	17.58	7.35	29.08	7.23	27.25	8.82	12.17
Females (n = 12)	2.92	1.16	16.33	5.94	30.92	6.27	27.92	9.11	11.08
Total (n = 24)	3.04	1.76	16.96	6.57	30.00	6.69	27.58	8.76	11.62
Fourth									
Males (n = 12)	3.83	1.47	26.50	1.62	39.92	9.92	36.00	7.86	16.67
Females (n = 12)	3.58	1.56	25.08	5.68	38.08	13.15	37.83	16.44	16.92
Total (n = 24)	3.71	1.49	25.79	4.15	39.00	11.43	36.92	12.63	16.79
Seventh									
Males (n = 12)	4.08	1.00	24.00	7.15	43.00	9.39	40.42	13.15	16.00
Females (n = 12)	4.17	1.47	25.75	4.88	41.33	12.38	42.42	13.10	18.33
Total (n = 24)	4.13	1.23	24.87	6.05	42.17	10.77	41.42	12.87	17.17
Combined Grade									
Males (n = 36)	3.69	1.67	22.69	6.96	37.33	10.57	34.56	11.36	14.94
Females (n = 36)	3.56	1.46	22.39	6.90	36.78	11.59	36.06	14.23	15.44
Total (N = 72)	3.63	1.56	22.54	6.88	37.06	11.02	35.31	12.81	15.19

TABLE 18

MEANS AND STANDARD DEVIATIONS FOR THE SOME-ALL, DICHOTOMOUS SORTING, CARDINALITY, AND ARITHMETIC ADDITION TASKS FOR THE LONGITUDINAL AND TESTING CONTROL SUBJECTS

Assessment Condition and Grade	Some-All		Dichotomous Sorting		Cardinality		Arithmet Additio	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	
Longitudinal Sample								
First								
Males (n = 26)	3.58	.64	1.77	.91	3.04	1.08	13.54	3
Females (n = 22)	3.73	.63	1.82	1.05	3.00	1.48	14.00	3
Total (n = 48)	3.65	.64	1.79	.97	3.02	1.26	13.75	3
Fourth								
Males (n = 26)	3.89	.33	2.77	.51	4.23	.91	15.62	1
Females (n = 28)	3.89	.32	2.71	.53	3.71	.90	15.82	
Total (n = 54)	3.89	.32	2.74	.52	3.96	.93	15.72	
Combined Grade								
Males (n = 52)	3.73	.53	2.27	.89	3.64	1.15	14.58	2
Females (n = 50)	3.82	.48	2.32	.91	3.40	1.23	15.02	2
Total (N = 102	3.78	.51	2.29	.90	3.52	1.19	14.79	2
Testing Control Sample								
First								
Males (n = 12)	3.67	.65	1.75	.87	2.50	1.17	13.00	5
Females (n = 12)	3.58	.67	2.25	.75	2.67	1.30	14.67	1
Total (n = 24)	3.63	.65	2.00	.83	2.58	1.21	13.83	3
Fourth								
Males (n = 12)	4.00	.00	2.50	.67	4.08	1.16	15.58	
Females (n = 12)	4.00	.00	2.75	.45	4.25	.75	15.83	
Total (n = 24)	4.00	.00	2.63	.58	4.17	.96	15.71	
Combined Grade								
Males (n = 24)	3.83	.48	2.13	.85	3.29	1.40	14.29	3
Females (n = 24)	3.79	.51	2.50	.66	3.46	1.32	15.25	1
Total (N = 48)	3.81	.49	2.31	.78	3.38	1.35	14.77	2

TABLE 19

MEANS AND STANDARD DEVIATIONS FOR THE SERIATION, LENGTH TRANSITIVITY, AND WEIGHT TRANSITIVITY TASKS ADMINISTERED TO THE LONGITUDINAL AND TESTING CONTROL SUBJECTS

Assessment Condition and Grade	Seriation $\bar{X}$	Seriation S.D.	Length Transitivity $\bar{X}$	Length Transitivity S.D.	Weight Transitivity $\bar{X}$	Weight Transitivity S.D.
Longitudinal Sample						
First						
Males (n = 26)	3.23	1.07	3.77	2.12	4.89	.59
Females (n = 22)	2.96	1.21	3.68	2.10	4.23	1.77
Total (n = 48)	3.10	1.13	3.73	2.09	4.58	1.30
Fourth						
Males (n = 26)	4.46	.65	4.92	.39	4.81	.98
Females (n = 28)	3.96	.79	4.79	.63	4.93	.38
Total (n = 54)	4.20	.76	4.85	.53	4.87	.73
Combined Grade						
Males (n = 52)	3.85	1.07	4.35	1.62	4.85	.80
Females (n = 50)	3.52	1.11	4.30	1.56	4.62	1.24
Total (N = 102)	3.69	1.10	4.32	1.58	4.74	1.04
Testing Control Sample						
First						
Males (n = 12)	2.50	1.17	3.17	2.41	4.75	.62
Females (n = 12)	2.83	.72	3.50	2.07	4.25	1.60
Total (n = 24)	2.67	.96	3.33	2.20	4.50	1.22
Fourth						
Males (n = 12)	4.33	.78	4.83	.58	4.83	.39
Females (n = 12)	4.00	.85	4.83	.58	5.00	.00
Total (n = 24)	4.17	.82	4.83	.56	4.92	.28
Combined Grade						
Males (n = 24)	3.42	1.35	4.00	1.91	4.79	.51
Females (n = 24)	3.42	.97	4.17	1.63	4.63	1.17
Total (N = 48)	3.42	1.16	4.08	1.76	4.71	.90

TABLE 20

MEANS AND STANDARD DEVIATIONS FOR THE NUMBER CONSERVATION, LENGTH CONSERVATION, AND WEIGHT CONSERVATION TASKS ADMINISTERED TO THE LONGITUDINAL AND TESTING CONTROL SUBJECTS

Assessment Condition and Grade	Number Conservation		Length Conservation				Weight Conservation			
			Identity		Equivalence		Identity		Equivalence	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
Longitudinal Sample										
First										
Males (n = 26)	5.54	2.21	4.42	1.84	4.27	2.36	4.46	1.86	3.62	2.62
Females (n = 22)	5.32	2.21	4.50	1.77	3.77	2.58	4.36	1.84	4.00	2.54
Total (n = 48)	5.44	2.19	4.46	1.79	4.04	2.45	4.42	1.83	3.79	2.57
Fourth										
Males (n = 26)	6.89	.33	5.58	1.14	5.42	1.70	5.77	.86	5.62	1.39
Females (n = 28)	6.79	.50	5.04	1.62	4.54	2.46	5.11	1.66	4.61	2.42
Total (n = 54)	6.83	.42	5.30	1.42	4.96	2.15	5.43	1.37	5.09	2.04
Combined Grade										
Males (n = 52)	6.21	1.71	5.00	1.62	4.85	2.12	5.12	1.58	4.62	2.31
Females (n = 50)	6.14	1.67	4.80	1.69	4.20	2.52	4.78	1.76	4.34	2.47
Total (N = 102)	6.18	1.68	4.90	1.65	4.53	2.33	4.95	1.67	4.48	2.38
Testing Control Sample										
First										
Males (n = 12)	4.75	2.56	3.67	1.72	2.67	2.23	3.33	2.15	3.08	2.43
Females (n = 12)	4.83	2.79	3.58	1.68	2.33	2.64	4.17	1.80	3.08	2.23
Total (n = 24)	4.79	2.62	3.63	1.66	2.50	2.40	3.75	1.98	3.08	2.28
Fourth										
Males (n = 12)	6.67	.89	6.00	.00	5.17	1.99	4.92	1.68	4.67	2.46
Females (n = 12)	7.00	.00	5.67	1.15	5.50	1.73	5.33	1.56	4.50	2.71
Total (n = 24)	6.83	.64	5.83	.82	5.33	1.83	5.13	1.60	4.58	2.54
Combined Grade										
Males (n = 24)	5.71	2.11	4.83	1.69	3.92	2.43	4.13	2.05	3.88	2.52
Females (n = 24)	5.92	2.22	4.63	1.76	3.92	2.72	4.75	1.75	3.79	2.54
Total (N = 48)	5.81	2.15	4.73	1.71	3.92	2.55	4.44	1.91	3.83	2.50

TABLE 21

FACTORIAL VARIANCE ANALYSES SUMMARY FOR THE *GROUPEMENT* TASKS WITH THREE FACTORS: ASSESSMENT CONDITION (LONGITUDINAL VERSUS TESTING CONTROL SUBJECTS), GRADE LEVEL (FIRST, FOURTH, AND SEVENTH), AND SEX

	Sources of Variance[a]														
	Condition		Grade		Sex		Condition x Grade		Condition x Sex		Grade x Sex		Condition x Grade x Sex		Error
Variable	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS
Groupement I	3.34	1.46	34.10	14.94[d]	1.01	.44	1.53	.67	1.30	.57	.01	.00	.53	.23	2.28
Groupement II	6.52	2.69	32.58	13.43[d]	.74	.31	.41	.17	.09	.04	3.18	1.31	1.38	.57	2.42
Groupement III	8.76	1.78	14.20	2.88	4.25	.86	1.96	.40	.49	.10	9.00	1.82	.20	.04	4.93
Groupement IV	17.20	3.34	56.50	10.99[d]	2.83	.55	5.18	1.01	1.67	.32	16.13	3.14[b]	4.51	.88	5.14
Combined *Groupements* I-IV	131.98	3.89[b]	497.56	14.65[d]	.27	.01	20.41	.60	2.03	.06	70.01	2.06	11.15	.33	33.97
Groupement V	.28	.23	4.49	3.69[b]	2.29	1.88	1.21	.99	.02	.02	.23	.19	1.75	1.44	1.22
Groupement VI	.23	.17	15.68	11.48[d]	.03	.02	.56	.41	4.52	3.31	.46	.33	.78	.57	1.37
Groupement VII	2.52	1.08	14.74	6.33[c]	.22	.09	.31	.14	.01	.00	1.13	.49	2.34	1.01	2.33
Groupement VIII	.05	.03	33.18	17.20[d]	.03	.02	.85	.44	4.49	2.33	.02	.01	.76	.39	1.93
Combined *Groupements* V-VIII	3.47	.31	235.22	20.70[d]	5.41	.48	9.67	.85	19.97	1.76	.07	.01	14.79	1.30	11.36

a Degrees of Freedom: Assessment Condition = 1, Grade = 2, Sex = 1, Condition x Grade = 2, Condition x Sex = 1, Grade x Sex = 2, Condition x Grade x Sex = 2, Error = 216.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 22

FACTORIAL VARIANCE ANALYSES SUMMARY FOR CLASS INCLUSION, COMBINATORIAL REASONING TASK, AUDITORY MEMORY SPAN, AUDITORY SEQUENTIAL MEMORY, AND VISUAL ORIENTATION MEMORY WITH THREE FACTORS: ASSESSMENT CONDITION (LONGITUDINAL VERSUS TESTING CONTROL SUBJECTS), GRADE LEVEL (FIRST, FOURTH, AND SEVENTH), AND SEX

	Sources of Variance[a]														
	Condition		Grade		Sex		Condition x Grade		Condition x Sex		Grade x Sex		Condition x Grade x Sex		Error
Variable	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS
Class Inclusion	5.99	3.59	20.86	12.51[d]	.00	.00	.05	.03	.87	.52	.39	.23	.02	.01	1.67
Combinatorial Reasoning	50.32	1.07	1310.45	27.98[d]	.35	.01	31.55	.67	2.41	.05	37.00	.79	1.28	.03	46.84
Auditory Memory Span	41.93	.42	1951.77	19.69[d]	3.78	.04	108.69	1.10	34.11	.34	214.92	2.17	51.32	.52	99.11
Auditory Sequential Memory	58.39	.32	1770.04	9.71[d]	101.96	.56	243.56	1.34	.18	.00	20.90	.11	55.59	.30	182.35
Visual Orientation Memory	1.27	.14	551.13	61.12[d]	.22	.02	11.25	1.25	9.22	1.02	27.60	3.06	13.89	1.54	9.02

a Degrees of Freedom: Assessment Condition = 1, Grade = 2, Sex = 1, Condition x Grade = 2, Condition x Sex = 1, Grade x Sex = 2, Condition x Grade x Sex = 2, Error = 216.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 23

FACTORIAL VARIANCE ANALYSES
SUMMARY FOR THE SOME-ALL, DICHOTOMOUS SORTING, CARDINALITY, AND ARITHMETIC ADDITION TASKS WITH THREE FACTORS: ASSESSMENT CONDITION (LONGITUDINAL VERSUS TESTING CONTROL SUBJECTS), GRADE LEVEL (FIRST AND FOURTH), AND SEX

	Sources of Variance[a]														
	Condition		Grade		Sex		Condition x Grade		Condition x Sex		Grade x Sex		Condition x Grade x Sex		Error
Variable	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS
Some-All	.06	.24	3.05	12.86[d]	.01	.05	.16	.66	.12	.50	.01	.03	.10	.44	.24
Dichotomies	.07	.12	20.14	35.77[d]	1.13	2.00	.85	1.51	1.16	2.07	.25	.45	.04	.08	.56
Cardinality	.48	.39	52.37	43.21[d]	.10	.08	3.23	2.67	1.61	1.32	.46	.38	.46	.38	1.21
Arithmetic Addition	.02	.00	119.04	18.82[d]	13.59	2.15	.04	.01	3.17	.50	5.69	.90	2.74	.43	6.32

a Degrees of Freedom: Assessment Condition, Grade, Sex, Condition x Grade, Condition x Sex, Grade x Sex, Condition x Grade x Sex = 1; Error = 142.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 24

FACTORIAL VARIANCE ANALYSES SUMMARY FOR THE SERIATION, LENGTH TRANSITIVITY, AND WEIGHT TRANSITIVITY TASKS WITH THREE FACTORS: ASSESSMENT CONDITION (LONGITUDINAL VERSUS TESTING CONTROL SUBJECTS), GRADE LEVEL, AND SEX (FIRST AND FOURTH)

	Sources of Variance[a]														
	Condition		Grade		Sex		Condition x Grade		Condition x Sex		Grade x Sex		Condition x Grade x Sex		Error
Variable	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS
Seriation	1.82	2.11	55.88	64.87[d]	1.22	1.41	1.17	1.36	1.22	1.41	1.60	1.86	.40	.47	.86
Transitivity of Length	1.39	.58	56.25	23.64[d]	.02	.01	1.12	.47	.63	.27	.30	.13	.16	.07	2.38
Transitivity of Weight	.00	.00	4.32	4.50[b]	1.54	1.60	.09	.09	.08	.09	4.25	4.42[b]	.03	.03	.96

a Degrees of Freedom: Assessment Condition, Grade, Sex, Condition x Grade, Condition x Sex, Grade x Sex, Condition x Grade x Sex = 1; Error = 142.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 25

FACTORIAL VARIANCE ANALYSES SUMMARY FOR THE NUMBER CONSERVATION, LENGTH CONSERVATION, AND WEIGHT CONSERVATION TASKS WITH THREE FACTORS: ASSESSMENT CONDITION (LONGITUDINAL VERSUS TESTING CONTROL SUBJECTS), GRADE LEVEL (FIRST AND FOURTH), AND SEX

	Sources of Variance[a]														
Variable	Condition		Grade		Sex		Condition x Grade		Condition x Sex		Grade x Sex		Condition x Grade x Sex		Error
	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS	F value	MS
Number Conservation	3.32	1.17	96.80	34.26[d]	.02	.01	3.28	1.16	1.10	.39	.28	.10	.03	.01	2.83
Identity Conservation of Length	.78	.33	75.87	32.51[d]	1.58	.68	15.13	6.48[b]	.00	.00	1.53	.66	.28	.12	2.33
Equivalence Conservation of Length	11.09	2.18	117.03	23.01[d]	3.90	.77	28.61	5.63[b]	3.90	.77	.15	.03	2.28	.45	5.09
Identity Conservation of Weight	7.75	2.79	46.91	16.87[d]	.49	.18	.99	.36	8.22	2.96	1.96	.70	.04	.02	2.78
Equivalence Conservation of Weight	12.76	2.32	63.98	11.61[d]	1.27	.23	.31	.06	.42	.08	4.95	.90	3.06	.56	5.51

a Degrees of Freedom: Assessment Condition, Grade, Sex, Condition x Grade, Condition x Sex, Grade x Sex, Condition x Grade x Sex = 1; Error = 142.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 26

MEANS AND STANDARD DEVIATIONS FOR THE FOUR CLASS GROUPEMENT TASKS FOR SUBJECTS ASSESSED IN BOTH FIRST AND SECOND YEARS

Assessment Year and Grade	Groupements								Total Class	
	I		II		III		IV			
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
First Year										
Kindergarten										
Males (n = 26)	4.12	1.51	4.62	1.60	3.39	1.39	3.85	1.74	15.96	3.77
Females (n = 22)	4.64	1.36	4.73	1.86	3.23	1.85	3.64	2.19	16.23	5.22
Total (n = 48)	4.35	1.45	4.67	1.71	3.31	1.60	3.75	1.94	16.08	4.44
Third										
Males (n = 26)	4.50	1.30	4.81	1.42	3.12	2.22	4.58	2.18	17.00	4.99
Females (n = 28)	4.54	1.29	4.61	1.17	3.46	2.50	5.29	2.19	17.89	4.76
Total (n = 54)	4.52	1.29	4.70	1.28	3.30	2.35	4.94	2.19	17.46	4.85
Sixth										
Males (n = 26)	5.92	1.41	6.08	1.50	4.23	2.14	6.19	1.79	22.42	5.12
Females (n = 28)	6.39	1.23	6.68	1.34	5.00	2.52	6.39	2.27	24.46	5.45
Total (n = 54)	6.17	1.33	6.39	1.43	4.63	2.36	6.30	2.03	23.48	5.34
Combined Grade										
Males (n = 78)	4.85	1.60	5.17	1.62	3.58	1.98	4.87	2.13	18.46	5.41
Females (n = 78)	5.23	1.55	5.39	1.73	3.95	2.45	5.22	2.45	19.78	6.22
Total (N = 156)	5.04	1.58	5.28	1.68	3.76	2.23	5.05	2.30	19.12	5.85
Second Year										
First										
Males (n = 26)	4.73	.92	5.31	1.59	3.81	2.37	4.85	2.51	18.69	5.85
Females (n = 22)	4.82	1.22	5.05	1.46	3.64	2.22	4.09	2.20	17.59	5.21
Total (n = 48)	4.77	1.06	5.19	1.53	3.73	2.28	4.50	2.38	18.19	5.53
Fourth										
Males (n = 26)	5.81	1.74	5.65	1.60	4.27	2.16	6.50	1.77	22.23	6.03
Females (n = 28)	5.57	1.69	5.61	1.57	4.25	2.22	5.82	2.45	21.25	6.20
Total (n = 54)	5.69	1.70	5.63	1.57	4.26	2.17	6.15	2.16	21.72	6.10
Seventh										
Males (n = 26)	6.23	1.56	6.50	1.50	4.31	2.07	6.73	2.13	23.77	5.32
Females (n = 28)	6.32	1.66	6.57	1.64	5.68	2.16	6.89	1.91	25.46	6.05
Total (n = 54)	6.28	1.60	6.54	1.56	5.02	2.21	6.82	2.00	24.65	5.72
Combined Grade										
Males (n = 78)	5.59	1.57	5.82	1.63	4.13	2.19	6.03	2.29	21.56	6.07
Females (n = 78)	5.63	1.65	5.80	1.67	4.59	2.33	5.72	2.44	21.73	6.61
Total (N = 156)	5.61	1.60	5.81	1.64	4.36	2.27	5.87	2.37	21.65	6.33

TABLE 27

MEANS AND STANDARD DEVIATIONS FOR THE FOUR RELATIONS GROUPEMENT TASKS FOR SUBJECTS ASSESSED IN BOTH FIRST AND SECOND YEARS

Assessment Year and Grade	*Groupements* V $\bar{X}$	V S.D.	VI $\bar{X}$	VI S.D.	VII $\bar{X}$	VII S.D.	VIII $\bar{X}$	VIII S.D.	Total Relations $\bar{X}$	Total Relations S.D.
First Year										
Kindergarten										
Males (n = 26)	6.42	1.58	6.65	1.79	5.50	2.16	5.23	1.88	23.81	5.37
Females (n = 22)	6.46	1.57	6.23	2.00	5.27	2.23	5.05	1.65	23.00	5.22
Total (n = 48)	6.44	1.56	6.46	1.88	5.40	2.17	5.15	1.76	23.44	5.26
Third										
Males (n = 26)	6.81	1.52	7.89	.43	6.19	2.17	6.35	1.47	27.23	4.17
Females (n = 28)	7.25	1.21	7.68	1.06	6.00	2.39	6.07	1.88	27.00	3.82
Total (n = 54)	7.04	1.37	7.78	.82	6.09	2.67	6.20	1.69	27.11	3.96
Sixth										
Males (n = 26)	7.73	.67	7.65	1.09	6.85	1.22	7.12	1.28	29.35	2.80
Females (n = 28)	7.00	1.76	7.61	1.20	6.82	1.47	6.86	1.43	28.29	3.95
Total (n = 54)	7.35	1.39	7.63	1.14	6.83	1.34	6.98	1.35	28.80	3.46
Combined Grade										
Males (n = 78)	6.99	1.42	7.40	1.33	6.18	1.96	6.23	1.73	26.79	4.78
Females (n = 78)	6.94	1.54	7.24	1.55	6.09	2.12	6.06	1.80	26.33	4.77
Total (N = 156)	6.96	1.48	7.32	1.44	6.14	2.04	6.15	1.76	26.56	4.76
Second Year										
First										
Males (n = 26)	7.50	.86	7.35	1.79	6.50	1.96	6.35	1.85	27.69	4.70
Females (n = 22)	6.82	1.74	6.77	1.72	6.18	2.02	5.86	1.73	25.64	4.74
Total (n = 48)	7.19	1.36	7.08	1.76	6.35	1.97	6.13	1.79	26.75	4.78
Fourth										
Males (n = 26)	7.39	1.13	7.89	.33	7.19	1.30	6.89	1.28	29.35	2.64
Females (n = 28)	7.18	1.28	7.43	1.26	7.18	1.19	6.75	1.46	28.54	2.84
Total (n = 54)	7.28	1.20	7.65	.95	7.19	1.23	6.82	1.36	28.93	2.75
Seventh										
Males (n = 26)	7.54	.99	7.89	.43	7.08	1.92	7.58	.81	30.08	3.07
Females (n = 28)	7.71	.66	7.93	.38	7.18	1.44	7.21	1.20	30.04	2.73
Total (n = 54)	7.63	.83	7.91	.40	7.13	1.67	7.39	1.04	30.06	2.87
Combined Grade										
Males (n = 78)	7.47	.99	7.71	1.09	6.92	1.76	6.94	1.45	29.04	3.67
Females (n = 78)	7.27	1.30	7.42	1.27	6.90	1.59	6.67	1.53	28.26	3.83
Total (N = 156)	7.37	1.15	7.56	1.19	6.91	1.67	6.80	1.50	28.65	3.76

TABLE 28

MEANS AND STANDARD DEVIATIONS FOR CLASS INCLUSION, COMBINATORIAL REASONING, AUDITORY MEMORY SPAN TEST, AUDITORY SEQUENTIAL MEMORY TEST, AND VISUAL ORIENTATION MEMORY FOR SUBJECTS ASSESSED IN BOTH FIRST AND SECOND YEARS

Assessment Year and Grade	CI $\bar{X}$	CI S.D.	CR $\bar{X}$	CR S.D.	AMS $\bar{X}$	AMS S.D.	ASM $\bar{X}$	ASM S.D.	VOM $\bar{X}$	VOM S.D.
First Year										
Kindergarten										
Males (n = 26)	2.04	1.40	7.12	5.85	23.77	5.61	20.96	9.84	9.00	3.27
Females (n = 22)	3.09	1.44	8.50	7.87	25.27	10.48	24.73	14.60	8.64	2.19
Total (n = 48)	2.52	1.50	7.75	6.81	24.46	8.15	22.69	12.26	8.83	2.81
Third										
Males (n = 26)	3.54	1.14	19.62	8.09	31.31	8.46	30.96	10.79	13.73	2.95
Females (n = 28)	3.82	1.22	19.25	8.75	30.79	10.17	27.64	10.78	13.32	3.64
Total (n = 54)	3.69	1.18	19.43	8.36	31.04	9.30	29.24	10.81	13.52	3.30
Sixth										
Males (n = 26)	4.19	1.13	23.69	6.49	34.50	9.68	34.69	9.86	17.04	1.73
Females (n = 28)	4.50	.88	25.00	6.63	36.68	10.25	36.32	12.64	17.86	2.03
Total (n = 54)	4.35	1.01	24.37	6.53	35.63	9.94	35.54	11.31	17.46	1.92
Combined Grade										
Males (n = 78)	3.26	1.52	16.81	9.82	29.86	9.18	28.87	11.61	13.26	4.28
Females (n = 78)	3.86	1.30	18.28	10.16	31.35	11.14	29.94	13.39	13.63	4.59
Total (N = 156)	3.56	1.44	17.55	9.99	30.60	10.20	29.40	12.50	13.44	4.43
Second Year										
First										
Males (n = 26)	3.31	1.41	17.04	9.25	28.38	8.66	27.88	12.84	12.27	4.68
Females (n = 22)	3.36	1.18	16.55	8.90	34.95	12.03	32.68	16.78	12.41	3.66
Total (n = 48)	3.33	1.29	16.81	9.00	31.40	10.75	30.08	14.81	12.33	4.20
Fourth										
Males (n = 26)	4.08	1.13	23.50	6.13	37.54	10.76	36.73	16.20	17.00	2.17
Females (n = 28)	4.14	1.08	22.86	7.32	33.50	11.20	35.68	15.72	15.00	3.29
Total (n = 54)	4.11	1.09	23.17	6.72	35.44	11.08	35.94	15.80	15.96	2.96
Seventh										
Males (n = 26)	4.35	1.32	23.85	6.21	40.81	8.61	36.46	12.86	17.35	2.28
Females (n = 28)	4.61	.79	25.39	5.36	41.61	7.96	36.36	10.15	18.11	1.52
Total (n = 54)	4.48	1.08	24.65	5.78	41.22	8.21	36.41	11.42	17.74	1.94
Combined Grade										
Males (n = 78)	3.91	1.35	21.46	7.90	35.58	10.68	33.53	14.44	15.54	3.97
Females (n = 78)	4.09	1.12	21.99	7.96	36.82	10.91	35.08	14.21	15.38	3.68
Total (N = 156)	4.00	1.24	21.72	7.91	36.20	10.78	34.30	14.30	15.46	3.82

TABLE 29

MEANS AND STANDARD DEVIATIONS FOR THE SOME-ALL, DICHOTOMOUS SORTING, CARDINALITY AND ARITHMETIC ADDITION TASKS FOR SUBJECTS ASSESSED IN BOTH FIRST AND SECOND YEARS

Assessment Year and Grade	Some-All		Dichotomous Sorting		Cardinality		Arithmetic Addition	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
First Year								
Kindergarten								
Males (n = 26)	3.54	.58	1.12	.71	1.92	1.32	4.96	5.16
Females (n = 22)	3.73	.63	.95	.95	1.82	1.22	6.41	5.47
Total (n = 48)	3.63	.61	1.04	.82	1.88	1.27	5.63	5.30
Third								
Males (n = 26)	3.81	.49	2.04	.77	4.00	.89	15.77	.51
Females (n = 28)	3.86	.45	2.04	.79	3.57	1.10	15.54	.92
Total (n = 54)	3.83	.47	2.04	.78	3.78	1.02	15.65	.76
Combined Grade								
Males (n = 52)	3.67	.55	1.58	.87	2.96	1.53	10.37	6.55
Females (n = 50)	3.80	.53	1.56	1.01	2.80	1.44	11.52	5.85
Total (N = 102)	3.74	.54	1.57	.94	2.88	1.48	10.93	6.22
Second Year								
First								
Males (n = 26)	3.58	.64	1.77	.91	3.04	1.08	13.54	3.40
Females (n = 22)	3.73	.63	1.82	1.05	3.00	1.48	14.00	3.25
Total (n = 48)	3.65	.64	1.79	.97	3.02	1.26	13.75	3.30
Fourth								
Males (n = 26)	3.89	.33	2.77	.51	4.23	.91	15.62	1.24
Females (n = 28)	3.89	.32	2.71	.53	3.71	.90	15.82	.61
Total (n = 54)	3.89	.32	2.74	.52	3.96	.93	15.72	.96
Combined Grade								
Males (n = 52)	3.73	.53	2.27	.89	3.64	1.15	14.58	2.74
Females (n = 50)	3.82	.48	2.32	.91	3.40	1.23	15.02	2.36
Total (N = 102)	3.78	.51	2.29	.90	3.52	1.19	14.79	2.56

TABLE 30

MEANS AND STANDARD DEVIATIONS FOR THE SERIATION, LENGTH TRANSITIVITY, AND WEIGHT TRANSITIVITY TASKS FOR SUBJECTS ASSESSED IN BOTH FIRST AND SECOND YEARS

Assessment Year and Grade	Seriation $\bar{X}$	S.D.	Length Transitivity $\bar{X}$	S.D.	Weight Transitivity $\bar{X}$	S.D.
First Year						
Kindergarten						
Males (n = 26)	1.81	1.20	3.73	1.91	4.42	1.03
Females (n = 22)	1.96	.95	3.55	2.09	4.50	1.26
Total (n = 48)	1.88	1.08	3.65	1.97	4.46	1.13
Third						
Males (n = 26)	3.85	.88	4.19	1.83	4.62	1.36
Females (n = 28)	3.64	.68	4.46	1.37	4.93	.38
Total (n = 54)	3.74	.78	4.33	1.60	4.78	.98
Combined Grade						
Males (n = 52)	2.83	1.46	3.96	1.87	4.52	1.20
Females (n = 50)	2.90	1.17	4.06	1.77	4.74	.90
Total (N = 102)	2.86	1.32	4.01	1.81	4.63	1.06
Second Year						
First						
Males (n = 26)	3.23	1.07	3.77	2.12	4.89	.59
Females (n = 22)	2.96	1.21	3.68	2.10	4.23	1.77
Total (n = 48)	3.10	1.13	3.73	2.09	4.58	1.30
Fourth						
Males (n = 26)	4.46	.65	4.92	.39	4.81	.98
Females (n = 28)	3.96	.79	4.79	.63	4.93	.38
Total (n = 54)	4.20	.76	4.85	.53	4.87	.73
Combined Grade						
Males (n = 52)	3.85	1.07	4.35	1.62	4.85	.80
Females (n = 50)	3.52	1.11	4.30	1.56	4.62	1.24
Total (N = 102)	3.69	1.10	4.32	1.58	4.74	1.04

TABLE 31

MEANS AND STANDARD DEVIATIONS FOR THE NUMBER CONSERVATION, LENGTH CONSERVATION, AND WEIGHT CONSERVATION TASKS FOR SUBJECTS ASSESSED IN BOTH FIRST AND SECOND YEARS

Assessment Year and Grade	Number Conservation		Length Conservation				Weight Conservation			
			Identity		Equivalence		Identity		Equivalence	
	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.	$\bar{X}$	S.D.
First Year										
Kindergarten										
Males (n = 26)	4.12	2.27	3.08	1.83	1.85	2.07	3.39	1.70	1.92	2.31
Females (n = 22)	4.00	2.41	3.82	1.62	2.55	2.41	3.82	2.02	2.55	2.37
Total (n = 48)	4.06	2.31	3.42	1.76	2.17	2.23	3.58	1.84	2.21	2.33
Third Grade										
Males (n = 26)	6.85	.46	5.27	1.49	5.15	2.05	5.42	1.45	5.19	2.00
Females (n = 28)	6.57	1.07	4.75	1.76	3.89	2.77	4.39	2.01	3.86	2.76
Total (n = 54)	6.70	.84	5.00	1.64	4.50	2.51	4.89	1.82	4.50	2.49
Combined Grade										
Males (n = 52)	5.48	2.13	4.17	1.99	3.50	2.64	4.40	1.87	3.56	2.70
Females (n = 50)	5.44	2.19	4.34	1.75	3.30	2.67	4.14	2.01	3.28	2.65
Total (N = 102)	5.46	2.15	4.26	1.87	3.40	2.65	4.28	1.94	3.42	2.67
Second Year										
First										
Males (n = 26)	5.54	2.21	4.42	1.84	4.27	2.36	4.46	1.86	3.62	2.62
Females (n = 22)	5.32	2.21	4.50	1.77	3.77	2.58	4.36	1.84	4.00	2.54
Total (n = 48)	5.44	2.19	4.46	1.79	4.04	2.45	4.42	1.83	3.79	2.57
Fourth										
Males (n = 26)	6.89	.33	5.58	1.14	5.42	1.70	5.77	.86	5.62	1.39
Females (n = 28)	6.79	.50	5.04	1.62	4.54	2.46	5.11	1.66	4.61	2.42
Total (n = 54)	6.83	.42	5.30	1.42	4.96	2.15	5.43	1.37	5.09	2.04
Combined Grade										
Males (n = 52)	6.21	1.71	5.00	1.62	4.85	2.12	5.12	1.58	4.62	2.31
Females (n = 50)	6.14	1.67	4.80	1.69	4.20	2.52	4.78	1.76	4.34	2.47
Total (N = 102)	6.18	1.68	4.90	1.65	4.53	2.33	4.95	1.67	4.48	2.38

TABLE 32

FACTORIAL VARIANCE ANALYSES SUMMARY (MIXED MODEL) FOR THE *GROUPEMENT* TASKS WITH THREE FACTORS: GRADE LEVEL (KINDERGARTEN-FIRST/THIRD-FOURTH/SIXTH-SEVENTH), SEX, AND ASSESSMENT YEAR

Variable	Sources of Variance[a]															
	Grade		Sex		Grade x Sex		Ss/Grade x Sex (Between Group Error)	Year		Year x Grade		Year x Sex		Year x Grade x Sex		Ss x Year/Grade x Sex (Within Group Error)
	MS	F value	MS	F value	MS	F value		MS	F value	MS	F value	MS	F value	MS	F value	
Groupement I	146.53	27.06[d]	3.77	.70	2.71	.50	5.41	12.69	18.34[d]	4.01	5.79[c]	1.25	1.81	.02	.03	.69
Groupement II	142.60	23.44[d]	.40	.07	3.40	.56	6.08	11.04	13.84[d]	2.05	2.56	.58	.73	.43	.54	.80
Groupement III	97.12	7.92[d]	22.20	1.81	21.15	1.73	12.26	13.86	8.22[c]	1.38	.82	.06	.03	.81	.48	1.69
Groupement IV	301.99	23.62[d]	.99	.08	5.95	.47	12.79	26.67	20.21[d]	1.68	1.27	4.25	3.22	1.56	1.18	1.32
Combined *Groupements* I-IV	2520.65	28.97[d]	39.58	.45	78.52	.90	87.02	248.78	34.35[d]	34.20	4.72[b]	13.74	1.90	2.03	.28	7.24
Groupement V	23.98	6.18[c]	3.73	.96	3.10	.80	3.88	6.56	9.40[c]	.97	1.39	.16	.23	2.77	3.97[b]	.70
Groupement VI	64.59	14.75[d]	11.21	2.56	3.31	.76	4.38	2.31	5.16[b]	1.79	3.99[b]	.10	.22	.10	.23	.45
Groupement VII	65.08	7.34[d]	1.75	.20	1.23	.14	8.86	23.46	21.11[d]	2.44	2.19	.06	.05	.06	.06	1.11
Groupement VIII	124.70	20.96[d]	12.28	2.06	.25	.04	5.95	16.67	20.28[d]	1.04	1.26	.06	.07	.15	.19	.82
Combined *Groupements* V-VIII	1002.89	21.73[d]	102.13	2.21	13.28	.29	46.16	169.27	45.06[d]	13.89	3.70[b]	.52	.14	4.38	1.17	3.76

a Degrees of Freedom: Sex, Year, Year x Sex = 1, Grade, Grade x Sex, Year x Grade, Year x Grade x Sex = 2; Ss/Grade x Sex (Between Group Error), Ss x Year/Grade x Sex (Within Group Error) = 150.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 33

FACTORIAL VARIANCE ANALYSES SUMMARY (MIXED MODEL) FOR CLASS INCLUSION, COMBINATORIAL REASONING, AUDITORY MEMORY SPAN, AUDITORY SEQUENTIAL MEMORY, AND VISUAL ORIENTATION MEMORY WITH THREE FACTORS: GRADE LEVEL (KINDERGARTEN-FIRST/THIRD-FOURTH/SIXTH-SEVENTH), SEX, AND ASSESSMENT YEAR

	Sources of Variance[a]														
Variable	Grade		Sex		Grade x Sex		Ss/Grade x Sex (Between Group Error)	Year		Year x Grade		Year x Sex		Year x Grade x Sex		Ss x Year/Grade x Sex (Within Group Error)
	MS	F value	MS	F value	MS	F value		MS	F value	MS	F value	MS	F value	MS	F value	
Class Inclusion	111.61	31.14[d]	16.83	4.70[b]	1.91	.53	3.58	7.63	15.05[d]	1.38	2.72	1.53	3.02	.80	1.57	.51
Combinatorial Reasoning	7993.65	64.99[d]	32.44	.26	50.29	.41	122.99	681.26	30.06[d]	244.42	10.78[d]	3.37	.15	3.79	.17	22.66
Auditory Memory Span	5539.45	19.21[d]	225.05	.78	403.61	1.40	288.42	1122.71	48.78[d]	33.31	1.45	1.09	.05	48.55	2.11	23.02
Auditory Sequential Memory	4691.86	9.59[d]	127.52	.26	489.80	1.00	489.12	935.41	21.29[d]	169.99	3.87[b]	4.42	.10	17.34	.39	43.93
Visual Orientation Memory	2518.87	104.94[d]	4.94	.21	53.78	2.24	24.00	159.01	62.65[d]	34.35	13.53[d]	1.69	.67	3.80	1.50	2.54

a Degrees of Freedom: Sex, Year, Year x Sex = 1, Grade, Grade x Sex, Year x Grade, Year x Grade x Sex = 2; Ss/Grade x Sex (Between Group Error), Ss x Year/Grade x Sex (Within Group Error) = 150.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 34

FACTORIAL VARIANCE ANALYSES SUMMARY (MIXED MODEL) FOR THE SOME-ALL, DICHOTOMOUS SORTING, CARDINALITY, AND ARITHMETIC ADDITION TASKS WITH THREE FACTORS: GRADE LEVEL (KINDERGARTEN-FIRST/THIRD-FOURTH), SEX, AND ASSESSMENT YEAR

	Sources of Variance[a]														
Variable	Grade		Sex		Grade x Sex		Ss/Grade x Sex (Between Group Error)	Year		Year x Grade		Year x Sex		Year x Grade x Sex		Ss x Year/Grade x Sex (Within Group Error)
	MS	F value	MS	F value	MS	F value		MS	F value	MS	F value	MS	F value	MS	F value	
Some-All	4.90	7.64[c]	.91	1.42	.50	.78	.64	.04	.37	.01	.08	.01	.10	.00	.00	.11
Dichotomies	96.22	58.28[d]	.18	.11	.02	.01	1.65	13.42	64.40[d]	.02	.08	.03	.15	.11	.52	.21
Cardinality	209.89	58.01[d]	8.22	2.27	4.07	1.12	3.62	10.36	29.85[d]	5.83	16.81[d]	.00	.00	.04	.11	.35
Arithmetic Addition	3611.12	154.11[d]	19.72	.84	23.72	1.01	23.43	380.48	101.55[d]	408.89	109.13[d]	.33	.09	3.21	.86	3.75

a Degrees of Freedom: Grade, Sex, Grade x Sex, Year, Year x Grade, Year x Sex, Year x Grade x Sex = 1; Ss/Grade x Sex (Between Group Error), Ss x Year/Grade x Sex (Within Group Error) = 98.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 35

FACTORIAL VARIANCE ANALYSES SUMMARY (MIXED MODEL) FOR THE SERIATION, TRANSITIVITY OF LENGTH, AND TRANSITIVITY OF WEIGHT TASKS WITH THREE FACTORS: GRADE LEVEL (KINDERGARTEN-FIRST/THIRD-FOURTH), SEX, AND ASSESSMENT YEAR

	Sources of Variance[a]														
Variable	Grade		Sex		Grade x Sex		Ss/Grade x Sex (Between Group Error)	Year		Year x Grade		Year x Sex		Year x Grade x Sex		Ss x Year/Grade x Sex (Within Group Error)
	MS	F value	MS	F value	MS	F value		MS	F value	MS	F value	MS	F value	MS	F value	
Seriation	226.56	89.17[d]	4.75	1.86	2.06	.81	2.54	17.29	68.75[d]	3.51	13.96[d]	.80	3.17	.03	.10	.25
Transitivity of Length	83.28	14.02[d]	.08	.01	1.05	.18	5.94	2.51	2.00	1.26	1.00	.19	.15	.41	.32	1.26
Transitivity of Weight	9.39	3.72	.04	.02	6.51	2.58	2.53	.30	.66	.00	.00	1.27	2.81	.46	1.03	.45

a Degrees of Freedom: Grade, Sex, Grade x Sex, Year, Year x Grade, Year x Sex, Year x Grade x Sex = 1; Ss/Grade x Sex (Between Group Error), Ss x Year/Grade x Sex (Within Group Error) = 98.

b $p < .05$
c $p < .01$
d $p < .001$

TABLE 36

FACTORIAL VARIANCE ANALYSES SUMMARY FOR THE NUMBER CONSERVATION, LENGTH CONSERVATION, AND WEIGHT CONSERVATION TASKS WITH THREE FACTORS: GRADE LEVEL (KINDERGARTEN-FIRST/THIRD-FOURTH), SEX, AND ASSESSMENT YEAR

Variable	Sources of Variance[a]														
	Grade		Sex		Grade x Sex		Ss/Grade x Sex (Between Group Error)	Year		Year x Grade		Year x Sex		Year x Grade x Sex		Ss x Year/Grade x Sex (Within Group Error)
	MS	F value	MS	F value	MS	F value		MS	F value	MS	F value	MS	F value	MS	F value	
Number Conservation	417.05	54.92[d]	3.22	.42	.01	.00	7.59	13.06	17.14[d]	9.86	12.93[d]	.01	.02	.12	.16	.76
Identity Conservation of Length	149.77	20.46[d]	.81	.11	22.32	3.05	7.32	10.68	12.17[d]	3.34	3.80	.66	.76	.65	.74	.88
Equivalence Conservation of Length	278.67	19.93[d]	27.75	1.98	34.97	2.50	13.99	32.41	17.18[d]	12.23	6.48[b]	.84	.44	3.90	2.06	1.89
Identity Conservation of Weight	140.95	18.88[d]	13.94	1.87	26.01	3.48	7.47	11.67	11.24[c]	.54	.52	.02	.02	1.28	1.23	1.04
Equivalence Conservation of Weight	335.30	24.07[d]	15.11	1.09	71.00	5.10[b]	13.93	28.59	14.39[d]	6.26	3.15	.02	.01	.50	.25	1.99

a Degrees of Freedom: Grade, Sex, Grade x Sex, Year, Year x Grade, Year x Sex, Year x Grade x Sex = 1; Ss/Grade x Sex (Between Group Error), Ss x Year/Grade x Sex (Within Group Error) = 98.

b $p < .05$
c $p < .01$
d $p < .001$

Year of assessment (longitudinal change comparisons) effects were significant for all the logical concept tasks and memory ability measures except length and weight transitivity and the some-all understanding tasks. Sex/year interactions were uniformly nonsignificant. There were a considerable number of significant grade level/year interactions. A chi-square analog to Scheffé's theorem was performed to determine which pair-wise grade level/year differences were significant at the .05 level. Comparison of performance differences between the kindergarten-first grade group and the third-fourth grade group were significant for *groupement* VI, combinatorial reasoning, arithmetic addition, cardinality, and conservation of number. Differences between the kindertarten-first grade group and the sixth-seventh grade group were significant for combined relations *groupements*, combinatorial reasoning, auditory sequential memory, and visual orientation memory. Differences between the third-fourth grade and sixth-seventh grade groups were significant for combined class *groupements*, *groupement* I, and visual orientation memory. All differences showed a greater increase in performance from the first to the second assessment years for the younger subsample of each comparison.

A comprehensive analysis of the conservation and transitive inference two-year performance patterns has been conducted. These results may be briefly summarized:

> Results indicated a lack of presentation order, selective survival, repeated measurement, sex, and content area significant main effects or interactions. Analyses of the longitudinal sample subjects' conservation task performances over the annual interval indicated significant grade level distinctions, year 1 versus year 2 differences, and type of conservation distinctions. Identity conservation scores were consistently superior to equivalence conservation scores and this superiority was most notable for the younger subjects. Transitive inference tasks were significantly less difficult than equivalence conservation tasks. Most importantly, evidence for a developmental mastery sequence (transitivity → conservation) was demonstrated. Pass/fail comparisons indicated a lack of regression effects and greater growth for the conservation abilities as contrasted with transitivity task mastery. Identity/equivalence conservation task distinctions were most apparent for the without verbal justification response criterion [Hooper & Toniolo, 1976, p. viii].

In accord with previous normative comparisons (e.g., Brainerd, 1973b; Gonchar, 1975; Toniolo & Hooper, 1975) the relative difficulties of the transitive inference, conservation, and class inclusion tasks were examined. While predictions from orthodox Piagetian theory would claim equivalent item difficulties and associated developmental synchrony (cf. Flavell, 1970, 1971; Hooper, 1973; Piaget, 1972; Pinard & Laurendeau, 1969; Wohlwill, 1973), recent speculations and empirical research by Brainerd (1973b) contends that these concept acquisitions emerge in a developmental sequence; i.e., transitivity → conservation → class inclusion (see also, Achenbach & Weisz, 1975). To evaluate these propositions

a series of mixed model variance analyses were conducted on the proportion of correct responses (note that the theoretical score ranges varied across the present three task formats) demonstrated by the longitudinal sample subjects for these respective tasks (transitive inference and equivalence conservation without explanation for the length and weight content domains, and class inclusion understanding). The between-subject factors in each instance were sex and grade level (kindergarten/first and third/fourth grades). The within-subjects variables were assessment year and the pair-wise task comparisons at issue. The length and weight content domains were analyzed separately.

Considering initially the transitivity/conservation relationship in both the length and weight content cases the main effects, in the expected direction, of grade level and assessment year were significant (F values exceeded 15.90, df 1, 98, $p < .01$). More importantly, there were notable distinctions between the transitive inference and conservation task difficulties with the former task consistently less difficult (F values of 19.12 and 72.61, df 1, 98, $p < .01$ for the length and weight cases respectively). In the weight concept task comparisons significant grade/task-type and assessment year/task-type interactions were also found which reflected the greater transitivity/conservation difficulty distinctions at the younger age ranges.

In similar fashion, comparisons of the transitivity/class inclusion task cases also indicated significant grade level and assessment year main effects (F values exceeded 12.07, df 1, 98, $p < .01$). Task-type distinctions demonstrating the much greater difficulty of the class inclusion measure were highly significant. The greater relative task differences for the younger contrasted with the older subsample patterns was indicated by significant grade level/task interactions (both length and weight content cases) and a significant assessment year/task interaction effect for the weight content area.

In contrast to these task-type distinctions in relative difficulty, the comparisons between equivalence conservation and class inclusion failed to reveal a potential developmental sequence or hierarchy. Grade level and assessment year main effects were significant in addition to certain higher order interactions but there were no clear-cut performance superiorities shown for the conservation cases when compared to class inclusion understanding.

We may therefore conclude that Brainerd's (1973b) predicted disparity between transitivity and conservation, and between transitivity and class inclusion is essentially replicated. Moreover the greater distinctions present for the younger subjects are in accord with the developmental expectations. No significant differences, however, were observed for the conservation/class inclusion comparisons. Thus the present data suggest the following developmental sequence pattern: Transitive inference mastery → conservation mastery = class inclusion understanding.

Dimensional Analyses

Initially, we shall briefly review a previous analysis of a portion of the present subjects' first year performances. Dihoff (1975) performed parametric and nonmetric internal analyses on the current

year one *groupement* data in an attempt to assess the evidence for class and relational dimensions independent of difficulty among the eight tasks. While a varimax rotation of the first two principal components of correlations among the internal scores led to a first factor representing the four class *groupements* and a second representing the four relational tasks, *groupement* four, with the smallest loading on factor one, also displayed a significant loading on the relational task factor.

Clustering the gamma coefficients obtained from dichotomous scoring led to three clusters: {I, II}, {III, IV, V} and {VI, VII, VIII} in which both the class and the relational tasks contributed to a middle cluster. These same clusters also appeared in a two dimensional multidimensional scaling representation with tasks four and five not clearly separating along a class/relation dimension. When grade six subjects were removed from the analysis, tasks four and five separated in the hypothesized direction, but remained very close to one another.

Dihoff did not attempt to fit other tasks into the internal structure of the *groupements*, but did examine gamma coefficients among the *groupements*, class inclusion and combinatorial reasoning tasks. For both Piagetian tasks, *groupements* four and six showed the strongest relationships, with five and seven showing weak relations, and the first three class *groupements* displaying intermediate values of gamma. *Groupement* eight related quite strongly to class inclusion, but weakest of all to combinatorial reasoning.

The fact that *groupements* four and five were at opposite extremes in their relationship to the Piagetian tasks, but nearly indistinguishable in the internal scaling of the *groupements*, suggests that the two dimensions spanned by the *groupements* do not contain some important source of variance related to the Piagetian task performance. Noting that "although the class inclusion task is considered an operationalization of *groupements* I and II, it is most highly related to relational tasks [p. 39]", she inferred that these *groupements* cannot be considered alternate measurements of the ability measured by the traditional class inclusion task. Dihoff concluded that "traditional concrete level performance is not completely subsumed by individually operationalizing each of the *groupements*." Although the class *groupements* were not proposed as alternate measures of class inclusion but as measures of skills necessary to its development, the fact that, with the exception of *groupement* IV, they do not demonstrate strong relations to traditional Piagetian tasks is difficult to reconcile with the simultaneous acceptance of Brainerd's analysis and of his operationalization of the tasks.

Nonmetric scaling and clustering methods, by employing only rank order of similarity, are analogous to partialling out the first principal component of common variance, and dealing only with second and higher order resemblances and differences. The appropriate parametric comparison of a two-dimensional nonmetric solution could be obtained by ignoring the first principal component, and with it the origin, and plotting the loadings of the second and third components. Since the relationships of interest include the common variance among tasks, a parametric approach was adopted for the remaining analyses.

By including the Piagetian tasks, the *groupement* measures, and memory tests in a single analysis, it was possible to investigate the degree to which certain other sources of variance, different from the class-relation distinction, were contributing to covariance. In particular, the memory components of both the conventional Piagetian tasks and the *groupements* could be estimated. Furthermore, it was hoped that some light could be shed on the apparent failure of *groupement* IV to cluster with the other class *groupements* or their relational counterparts.

Total subject sample analysis--year one assessments. Since most subjects master these concrete reasoning tasks by grade four, the study of relationships among tasks in the full sample was restricted to those tasks that would not be expected to reach ceiling: the class *groupements*, the relational *groupements*, the memory tests, and the near formal stage class inclusion and combinatorial reasoning tasks. Inspection of the correlation matrix (see Table 37) reveals several high correlations consistent with expectations, and some that might be less expected. The relationship between the two auditory memory span tests (r = .72) suggests that they largely measure a single ability, to the limit of their reliabilities. Similarly, *groupements* I and II (r = .69) are parallel and in fact interchangeable tasks, the meaning of primary and secondary class having been abstracted to the point at which they are represented only by the attribute that the examiner mentions first. Perhaps less expected is the correlation (r = .68) between visual memory and combinatorial reasoning, although the tasks as administered clearly involve a visualization component and a significant memory load. Class inclusion and combinatorial reasoning intercorrelate at .49, reflecting overlap in logical demands and a shared visualization component, the correlation of class inclusion with visual memory being .48. The *groupements* with correlations greater than .30 with class inclusion are IV (r = .40), VI (r = .34), and VIII (r = .38). In the case of combinatorial reasoning, *groupement* I (r = .35) joins the list in addition to IV (r = .45), VI (r = .39), and VIII (r = .35). The correlation of each *groupement* with visual memory tends to be similar to the correlation with combinatorial reasoning, with *groupements* I, II, IV, and VIII showing the strongest relations to visual memory. The auditory memory span tasks show their highest relationship ($.37 \leq r \leq .39$) with combinatorial reasoning and visual memory, suggesting that the strong relation between the latter two tasks is in fact partly due to memory load, and not only to a shared visualization component. Class inclusion and *groupements* II, IV, and VIII also display moderate correlations with the auditory memory span tasks.

Principal components analysis led to the results presented in Table 38, with the first component accounting for 36% of the total variance, and the remaining two components with eigenvalues greater than one representing 12% and 9% of the total variance, respectively. Loadings on the first principal component are above .5, with the exception of *groupements* III, V, and VII. Visual memory and combinatorial

TABLE 37

INTERCORRELATIONS AMONG TASKS ADMINISTERED TO YEAR ONE
TOTAL SUBJECT SAMPLE
(COMBINING KINDERGARTEN, THIRD, AND SIXTH GRADES)

Task	GI	GII	GIII	GIV	CI	CR	GV	GVI	GVII	GVIII	ASM	AMS	VM
GI	1.000												
GII	.691												
GIII	.333	.322											
GIV	.411	.386	.368										
CI	.271	.223	.163	.396									
CR	.347	.297	.192	.445	.488								
GV	.161	.033	.208	.301	.275	.213							
GVI	.119	.040	.095	.329	.341	.387	.265						
GVII	.201	.126	.034	.203	.236	.206	.308	.260					
GVIII	.304	.140	.265	.261	.382	.352	.348	.444	.447				
ASM	.297	.312	.140	.299	.259	.367	.193	.145	.178	.282			
AMS	.232	.283	.202	.329	.315	.391	.167	.193	.151	.274	.721		
VM	.386	.355	.148	.394	.477	.675	.273	.321	.284	.418	.367	.375	1.000

Note: Value of correlation coefficient significant at the .05 level = .148, and at the .01 level = .193 (two-tailed probabilities).

TABLE 38

UNROTATED PRINCIPAL COMPONENTS FACTOR MATRIX OF TASKS ADMINISTERED TO YEAR ONE TOTAL SUBJECT SAMPLE (COMBINING KINDERGARTEN, THIRD, AND SIXTH GRADES)

	Factor		
Task	one	two	three
GI	.621	.474	-.331
GII	.550	.635	-.212
GIII	.420	.305	-.420
GIV	.673	.123	-.189
CI	.644	-.198	.023
CR	.730	-.062	.147
GV	.457	-.383	-.212
GVI	.503	-.499	-.073
GVII	.446	-.407	-.166
GVIII	.632	-.386	-.153
ASM	.604	.199	.594
AMS	.614	.168	.604
VM	.744	-.071	.105
Eigenvalues	4.619	1.562	1.219

reasoning exhibit the highest loadings on this general ability factor, .74 and .73 respectively. *Groupement* IV, class inclusion, and *groupements* I and VIII with loadings ranging from .67 to .62, also are heavily saturated with this common individual difference related dimension.

The second component contrasts *groupements* I-III with the relations *groupements* V-VIII. *Groupement* IV, which related strongly to the general component with the Piagetian tasks and visual memory, has only a slight positive loading on this second dimension of variance, appearing only slightly more like the class than the relations *groupements*. The third component is characterized by positive loadings of about .60 on the auditory memory tasks, comparable to their loadings on the first component. The *groupements* all show small negative loadings on this component, the largest negative loadings being exhibited by *groupements* I and III.

Varimax rotation led to a description of the structure displayed in Table 39. The first rotated factor is defined by the four relational *groupements*, with loadings ranging from .62 to .72. The Piagetian tasks and visual memory display moderate loadings ranging from .34 to .55 on this factor and on factor three defined by the auditory memory tasks. Factor two is most strongly defined by *groupements* one and two. The loadings of the other two class *groupements* on this factor are .65 and .54 for *groupements* III and IV, respectively. No other task has a loading of as much as .3 on this factor, suggesting that in the total subject sample for year one, performance on the class *groupements* is relatively independent of the other nine tasks. Approximately 40% of the total variance can be represented in the two dimensions characterized by factors one and three. Figure 1 locates the tasks in the plane of factors one and three.

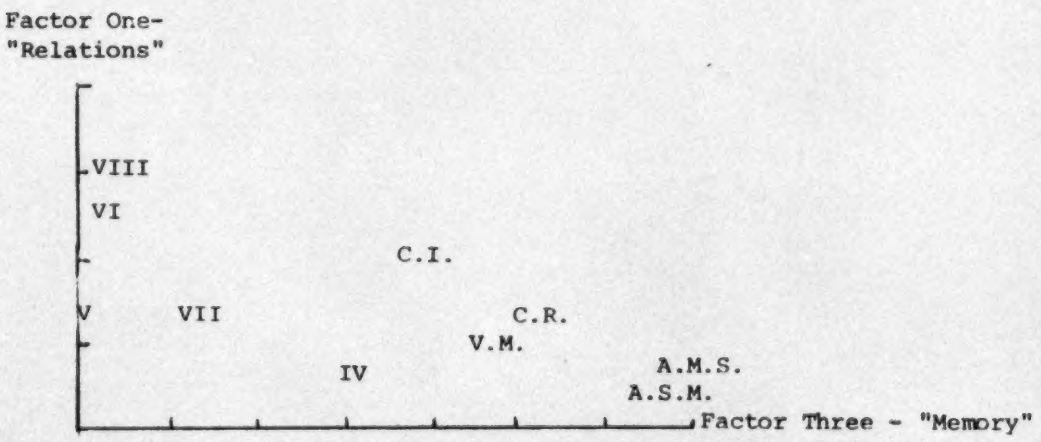

Figure 1. Representation of year one task performances in the plane of factors one and three.

TABLE 39

VARIMAX ROTATED FACTOR MATRIX, PRINCIPAL COMPONENTS PROCEDURE, OF TASKS ADMINISTERED TO YEAR ONE TOTAL SUBJECT SAMPLE (COMBINING KINDERGARTEN, THIRD, AND SIXTH GRADES)

	Factor		
Task	one	two	three
GI	.133	.818	.182
GII	-.060	.821	.269
GIII	.153	.649	-.038
GIV	.385	.540	.254
CI	.545	.200	.343
CR	.467	.268	.518
GV	.623	.107	.012
GVI	.701	-.020	.126
GVII	.622	.060	.037
GVIII	.721	.166	.157
ASM	.080	.130	.856
AMS	.106	.110	.864
VM	.494	.292	.490
Factor Variance	2.757	2.325	2.319

As is evident, the Piagetian tasks, figural memory, and, to a lesser extent, *groupement* IV, are in the interior of the quadrant defined by the relations *groupement* and the memory span tasks, suggesting that both relational operations and memory contribute to these composite tasks, with class inclusion slightly more dependent on logical operations, and combinatorial reasoning bearing a slightly heavier memory load. If the second principal component, representing class *groupements*, were included in this illustration, *groupement* IV would appear to be less strongly related to the other three tasks, appearing further out of the plane than would the CI, VM, CR cluster. However, class *groupements* I, II, and IV, with greater loadings on the memory factor than are shown by the relational *groupements*, would then appear nearer to the combinatorial reasoning task in the three dimensional space. The initial correlations among the class *groupements* and the Piagetian tasks, as high as those for the relational *groupements* in the case of combinatorial reasoning, would then be reproduced partly through shared dependency on the memory component.

The finding that the major common dimension underlying the relations *groupements* relates more strongly to class inclusion than does that underlying the first three class *groupements* suggests the existence of sources of variance other than the class/relation distinction distinguishing the tasks. One source of performance variance in the first three *groupements* may be the existence of bicolored figures in these tasks, coupled with the verbal switch from counting figures with "some red on them" to comparing figures with "red on them." If this format leads to the confusion among some children that the latter instruction refers to only the all red figures, the resulting decreased validity would lower the relationships between these class *groupements* and other tasks. *Groupement* IV utilizes a different stimulus set, with monocolored figures such as those used in the class inclusion assessment, and exhibits the strongest relationship of any of the *groupements* to both class inclusion and combinatorial reasoning. While *groupement* IV exhibits a slightly higher reliability than the other class *groupement* tasks, this argument depends not on reliability, but on the possibility that the first three *groupement* tasks are measuring, with reasonable reliability, something other than the first three *groupement* constructs.

Total subject sample analysis--year two assessments. As described above, 156 of the original longitudinal sample of 180 subjects initially assessed were re-tested one year later. The year one/year two intercorrelations for the subset of measures administered to all three age-grade levels are reported in Appendix A. The within-task test/re-test correlations are presented in Table 40. As can be seen, with the exception of *groupement* V, all of the values are significant and indicate a reasonable level of stability across the annual assessment interval. The correlations range from .179 to .660 with a median value of .506.

With an added year of maturity, the children came close to the ceiling of the two easiest *groupement* tasks, V and VI. Again, the highest intertask correlations (see Table 41) were between the pairs

TABLE 40

INTERCORRELATIONS BETWEEN INITIAL AND SECOND YEAR PERFORMANCES FOR THE MEASURES ADMINISTERED TO THE TOTAL SUBJECT SAMPLE (N = 156)

Tasks	Correlation Values
Groupement I	.424**
Groupement II	.413**
Groupement III	.343**
Groupement IV	.505**
Total Class *Groupements*	.595**
Groupement V	.179*
Groupement VI	.484**
Groupement VII	.368**
Groupement VIII	.396**
Total Relations *Groupements*	.595**
Class Inclusion	.416**
Combinatorial Reasoning	.389**
Auditory Sequential Memory (Digits)	.506**
Auditory Memory Span (Words)	.580**
Visual Orientation Memory	.660**

* probability value (two-tailed) < .05

** probability value (two-tailed) < .01

TABLE 41

INTERCORRELATIONS AMONG TASKS ADMINISTERED TO YEAR TWO TOTAL SUBJECT SAMPLE (COMBINING FIRST, FOURTH, AND SEVENTH GRADES)

Task	GI	GII	GIII	GIV	CI	CR	GV	GVI	GVII	GVIII	ASM	AMS	VM
GI	1.000												
GII	.774												
GIII	.507	.544											
GIV	.503	.488	.431										
CI	.412	.336	.335	.464									
CR	.275	.213	.307	.339	.288								
GV	.082	.143	.185	.223	.199	.096							
GVI	.210	.069	.154	.229	.310	.308	.161						
GVII	.208	.186	.279	.234	.330	.193	.322	.129					
GVIII	.317	.205	.253	.396	.421	.327	.275	.316	.419				
ASM	.324	.313	.267	.385	.305	.274	.058	.157	.197	.322			
AMS	.297	.284	.200	.260	.260	.199	.166	.180	.146	.235	.615		
VM	.411	.330	.258	.355	.349	.390	.046	.259	.219	.282	.316	.321	1.000

Note: Value of correlation coefficient significant at the .05 level = .159, and at the .01 level = .208 (two-tailed probabilities).

of parallel *groupement* tasks I and II (r = .77) and the two auditory memory tests (r = .62). *Groupement* IV again displayed the strongest relation to class inclusion (r = .46), followed by *groupements* VIII (r = .42) and I (r = 41) and visual memory (r = .35). In the case of combinatorial reasoning, the correlation with class inclusion was lower (r = .29) than in the previous year while visual memory again displayed the highest correlation (r = .39), although much reduced from the year one comparison value, followed by *groupements* IV (r = .34) and VIII (r = .33). The memory for digits test showed its strongest relations to class *groupements* I, II, IV and *groupement* VIII, followed by visual memory and class inclusion. These moderate correlations ranged from .31 to .39.

Principal components analysis led to the emergence of four components, accounting for 63% of the total variance (see Table 42). The first principal component accounted for 35% of the total variance and had its strongest loadings from the class *groupements* and class inclusion. The smallest loadings were due to the two relations *groupement* tasks that were approaching ceiling. The second component again contrasted class with relational *groupements*, while the third and fourth both contrasted the auditory memory tasks with other tasks: *Groupements* III, V, and VII in the case of component III, and combinatorial reasoning and *groupement* VI in the case of component IV. This last component accounts for less than 8% of total variance.

The first three components were subjected to a varimax rotation, resulting in the loadings given in Table 43. The first factor represents the class *groupements*, again with a moderate loading for *groupement* IV of .58, and small loadings for class inclusion, .36, and visual memory, .31. It can be identified with factor two of the year one analysis, but with the important emergence of relationships to class inclusion and visual memory. The second factor has its strongest loadings on the auditory memory tasks (.78 and .76) and visual memory (.57). Combinatorial reasoning, relational *groupements* VI and VIII, class inclusion and *groupement* IV showed loadings of over .3 on this factor. Thus factor two may be interpreted as growing out of year one factor three, but with the emergence of stronger relationships to certain of the *groupement* tasks (IV, VI, and VIII). Factor three is nearly identical to the first factor in the year one analysis, displaying strong loadings on all four relational *groupement* tasks and class inclusion, and to a lesser extent on *groupement* IV. The major difference is the lowered dependence on visual memory of this factor in year two balanced by the emergence of visual memory as a small but significant component of the class *groupement* factor. In year two, as in year one, it remains the case that the relational *groupement* factor is more strongly related to class inclusion than is the class *groupement* factor, while the memory factor is most strongly related to combinatorial reasoning. Thus, while the first three class *groupement* tasks are beginning to show relationships in the predicted directions, the possibility that they are not adequate measures of their constructs remains. Whereas the two dimensions most strongly related to the Piagetian tasks and visual memory in year one were the relational *groupements* and auditory memory, in year two the class *groupements* begin to emerge as significantly related to class inclusion. This

TABLE 42

UNROTATED PRINCIPAL COMPONENTS FACTOR MATRIX OF TASKS ADMINISTERED TO YEAR TWO TOTAL SUBJECT SAMPLE (COMBINING FIRST, FOURTH, AND SEVENTH GRADES

	Factor			
Task	one	two	three	four
GI	.741	-.433	-.184	-.088
GII	.683	-.530	-.276	.067
GIII	.639	-.247	-.338	-.013
GIV	.722	-.085	-.114	-.045
CI	.666	.170	-.067	-.100
CR	.543	.162	.188	-.433
GV	.325	.479	-.378	.397
GVI	.418	.407	.192	-.446
GVII	.475	.436	-.306	.247
GVIII	.612	.447	-.038	.004
ASM	.601	-.084	.530	.368
AMS	.536	-.050	.558	.461
VM	.603	-.047	.265	-.278
Eigenvalues	4.576	1.388	1.213	1.056

TABLE 43

VARIMAX ROTATED FACTOR MATRIX, PRINCIPAL COMPONENTS PROCEDURE, OF TASKS ADMINISTERED TO YEAR TWO FULL SAMPLE (COMBINING FIRST, FOURTH, AND SEVENTH GRADES)

	Varimax Factors		
Task	one	two	three
GI	.832	.265	.084
GII	.894	.153	.016
GIII	.719	.089	.242
GIV	.576	.326	.322
CI	.362	.342	.479
CR	.174	.477	.315
GV	.085	-.096	.679
GVI	-.060	.418	.446
GVII	.178	.049	.689
GVIII	.144	.346	.660
ASM	.212	.777	.016
AMS	.136	.763	.001
VM	.307	.565	.149
Factor Variance	2.724	2.364	2.091

necessitates three dimensions to describe the covariance of tasks defining the space in which the class inclusion task can be located (the memory and relational factors remain sufficient to locate combinatorial reasoning). Again *groupement* IV was separated from the first three class *groupements*. Of the class tasks it was most closely related to the Piagetian tasks.

The "migration" of visual memory from the relations to the class factor as the class factor begins to relate to Piagetian tasks seems reasonable if memory span operates as a threshold variable in the performance of these tasks. Under this assumption, a certain level of capacity would be necessary to successful manipulation of the variables but additional growth beyond the threshold would not lead to additional increments in performance. Examination of cross-sectional correlations in year one gives mixed evidence on this hypothesis. The third and sixth grade subsample intercorrelations among the memory task and combinatorial reasoning and class inclusion initial and second year performances are presented in Table 44. While combinatorial reasoning shows a lower relation to all memory tasks for the older age group in each of the six cross-sectional comparisons, class inclusion shows an increase for the auditory tasks from grade three to grade six, perhaps due as much to the emergence of verbal strategies for the task as to any memory capacity per se.

Summary of the two year total subject sample data. Changes from the initial to second assessment year are ideographically represented in Figure 2.

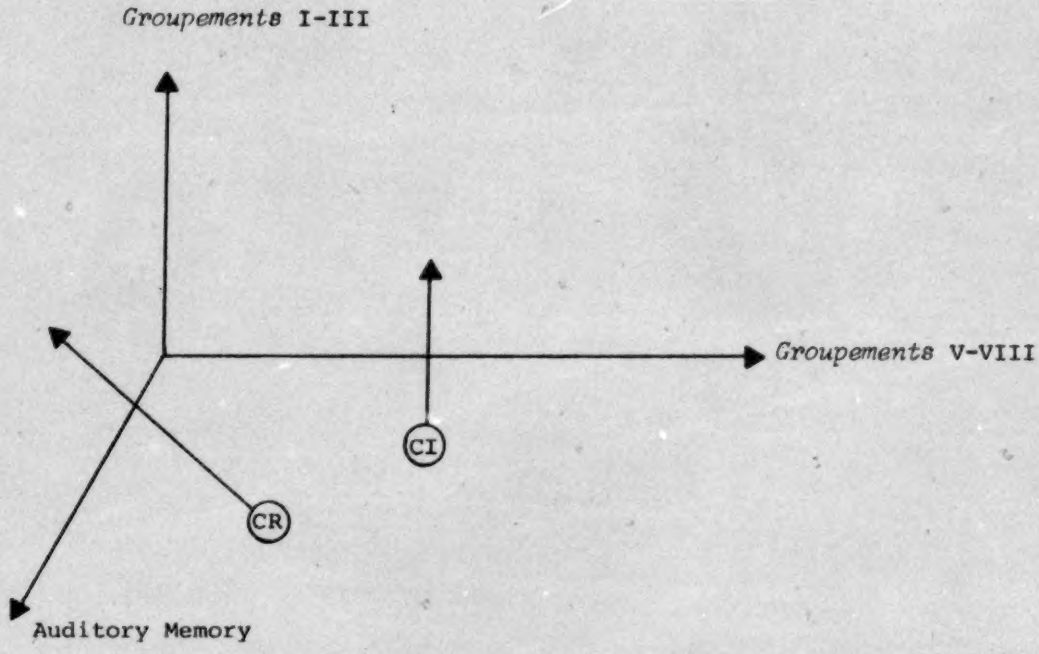

Figure 2. Year one to year two changes in location of Piagetian tasks in the space defined by *groupements* and memory factors.

TABLE 44

FIRST AND SECOND ASSESSMENT YEAR INTERCORRELATIONS AMONG THE MEMORY TASKS AND COMBINATORIAL REASONING, AND CLASS INCLUSION FOR THE THIRD/FOURTH, AND SIXTH/SEVENTH GRADE SUBJECTS

	Combinatorial Reasoning				Class Inclusion			
	Year One		Year Two		Year One		Year Two	
	3rd Grade	6th Grade	4th Grade	7th Grade	3rd Grade	6th Grade	4th Grade	7th Grade
Visual Orientation Memory	.447	.134	.287	.280	.203	.133	.293	.007
Auditory Memory Span	.224	.091	.116	.042	.095	.273	.253	.120
Auditory Sequential Memory	.222	.130	.361	.083	.104	.200	.429	.286

While the relationship of the relations *groupements* and memory span to class inclusion remained roughly constant, dependency of combinatorial reasoning and visual memory on the relational *groupements* decreased as the skills measured by the first three class *groupements* became more relevant to class inclusion performance and visual memory. In the absence of additional marker variables, this situation could equally well be represented by a figure in which the Piagetian tasks and figural memory remain constant in some psychological space, but the axes are rotated about the auditory memory axis indicating a change in what is being measured by the relational and the class *groupements*. Given the differential difficulty of the *groupements* this interpretation is attractive, but given the relatively stationary behavior of *groupement* IV and combinatorial reasoning, we have chosen the first representation, which suggests a development of strategy from a more perceptual to a more verbal or conceptual approach, and depicts active, figural memory as moving along a similar trajectory. The possibility that the meaning of the first three class *groupement* tasks changes with language maturation should not be discounted. However, the fact that the year one to year two predictability of the class *groupements* compares favorably with that of the relational *groupements* (see Table 40) does not argue for a basic change in the construct being measured over the annual assessment interval.

Complete assessment battery analysis--year one data. Because grade six children are at the upper limit of the concrete operational stage, the complete assessment battery was administered only to the 120 kindergarten and third grade children. In addition to the *groupements*, memory, class inclusion, and combinatorial reasoning tasks administered to the total subject sample, 11 concrete level Piagetian tasks and an arithmetic addition task were administered to this younger group in both years. Five of these tasks dealt with conservation, two with transitivity, and one each with the concepts, "some and all," cardinality, seriation, and dichotomous sorting. In the kindergarten-third grade sample, the some-all and transitivity of weight tasks approached ceiling and could be expected to have unstable correlations for this reason.

Inspection of the correlation matrix (see Table 45) reveals that the first three class *groupements* show few correlations above .10 with Piagetian tasks, the largest, (r = .126) being between equivalence conservation of weight and *groupement* I. *Groupement* IV displays correlations greater than .30 with cardinality, dichotomies, and combinatorial reasoning. Among the relations cases, *groupement* V correlates above .3 with class inclusion, dichotomies, and seriation; *groupement* VI above .4 with cardinality, addition, seriation, identity conservation of length, conservation of number; and above .3 with visual memory, dichotomies, and the remaining conservation tasks. *Groupement* VII displays no correlations above .3 with Piagetian tasks, while *groupement* VIII shows a similar but slightly weaker pattern to *groupement* VI.

Correlations greater than .60 appear between the two conservation of length tasks (r = .80), the two auditory memory tasks (r = .77), and appear within the series of number related tasks: conservation of

TABLE 45

INTERCORRELATIONS AMONG TASKS ADMINISTERED TO YEAR ONE KINDERGARTEN AND THIRD GRADE SUBJECT SAMPLE

Task	GI	GII	GIII	GIV	CI	CR	GV	GVI	GVII	GVIII	ASM	AMS	VM
GI	1.000												
GII	.541												
GIII	.168	.181											
GIV	.213	.243	.226										
CI	.030	.011	.011	.230									
CR	.088	.095	.066	.317	.396								
GV	.036	-.091	.074	.266	.322	.143							
GVI	.002	-.077	-.046	.213	.309	.399	.262						
GVII	.127	.006	-.085	.091	.167	.080	.284	.228					
GVIII	.147	-.094	.143	.103	.319	.201	.331	.456	.390				
ASM	.031	.035	-.020	.145	.117	.250	.183	.099	.095	.175			
AMS	.097	.089	.083	.272	.184	.319	.133	.188	.051	.158	.771		
VM	.063	.039	-.045	.213	.367	.577	.216	.339	.160	.246	.182	.248	
DICH	.039	.033	.114	.320	.367	.439	.300	.382	.239	.358	.253	.278	.497
SA	-.062	-.019	-.082	.014	.242	.320	.208	.132	.087	.045	.108	.131	.316
CARD	.114	.006	.064	.351	.357	.677	.227	.541	.113	.406	.338	.461	.596
TL	.048	.085	-.014	.044	.065	.080	.123	.107	.205	.179	.062	.078	.013
TW	.029	-.042	.035	.087	.281	.128	.158	.145	.021	.280	.186	.147	.086
SER	-.004	-.015	.076	.235	.437	.529	.316	.507	.180	.434	.270	.334	.613
CON NO	.009	-.099	-.023	.114	.293	.549	.234	.438	.180	.364	.153	.233	.561
ID CON LGTH	.111	.023	.074	-.004	.264	.358	.159	.422	.117	.302	.142	.215	.409
EQ CON LGTH	.116	.038	.098	.090	.213	.369	.126	.392	.102	.260	.185	.244	.461
ID CON WGT	.015	.003	.042	.097	.233	.268	.223	.323	.111	.283	.101	.157	.337
EQ CON WGT	.126	.112	.067	.060	.182	.288	.167	.378	.224	.322	.103	.152	.338
ARITH ADD	-.035	-.083	.019	.239	.464	.603	.240	.541	.113	.408	.333	.411	.564

TABLE 45, continued.

Task	DICH	SA	CARD	TL	TW	SER	CON NO	ID CON LGTH	EQ CON LGTH	ID CON WGT	EQ CON WGT	ARITH ADD
DICH	1.000											
SA	.245											
CARD	.573	.254										
TL	.106	-.047	.168									
TW	.169	-.021	.202	.204								
SER	.567	.321	.683	.071	.226							
CON NO	.525	.244	.715	.171	.027	.641						
ID CON LGTH	.311	.156	.425	.110	.090	.431	.523					
EQ CON LGTH	.339	.176	.444	.022	.028	.440	.514	.797				
ID CON WGT	.320	.161	.422	.017	.137	.458	.466	.416	.416			
EQ CON WGT	.336	.160	.457	.109	.129	.472	.394	.448	.451	.669		
ARITH ADD	.541	.362	.729	.147	.253	.743	.615	.452	.503	.496	.492	1.000

Note: Value of correlation coefficient significant at the .05 level = .178, and at the .01 level = .233 (two-tailed probabilities).

number, cardinality, seriation, addition, and combinatorial reasoning. Given the large and closely related set of tasks involving number, conservation, and relational thinking in the battery, we might expect relational *groupements* to show a stronger loading on the first principal component than class *groupements*, even more than was the case in the total subject sample analyses.

The first principal component (see Table 46), accounting for 31% of the total variance, displays its highest loadings of .85 on addition and cardinality, seriation .83, conservation of number .76, visual memory .71, and combinatorial reasoning .70. The other conservation tasks, dichotomies, and three of the four relational *groupements* have sizable loadings on this component. Of the relational *groupements*, only one has a loading smaller than .40 on the first component. *Groupement* IV loads significantly on this component, departing from the other class *groupements*.

The second component picks up the class *groupements*, with additional moderate loadings on the auditory memory tasks, and negligible loadings on all other variables.

The third component contrasts *groupements* I and II with auditory memory. The conservation of length and weight tasks have significant loadings on this component.

The fourth component is defined by *groupements* VII and VIII, with moderate loadings from *groupement* V, the transitivity tasks, and, as with the third component, negative relations to auditory memory span. The remaining three components, with eigenvalues of 1.33 and below, are essentially doublets, adjusting single correlations such as the two auditory tests, or *groupements* III and VII, higher or lower than the other components would predict.

Rotation of the first four components, accounting for 51.8% of the variance, led to the description presented in Table 47. The first factor is dominated by the conservation tasks with loadings ranging from .67 to .78, the number-related addition, seriation and cardinality tasks, and visual memory. Combinatorial reasoning, *groupements* VI and VIII, and all traditional Piagetian tasks with the exception of transitivity show significant amounts of variance in common with this factor.

Factor two is defined by auditory memory, with significant loadings on combinatorial reasoning, cardinality, arthmetic addition, dichotomies, seriation, visual memory, and class inclusion. Of the *groupements*, only case IV loads significantly on this memory span factor.

Factor three receives its highest loadings from the relations *groupements* (*groupement* VI displaying the lowest of these), from the transitivity tasks and class inclusion. Dichotomies displays a slight relationship to this factor.

Factor four isolates the class *groupements*, with no other tasks showing a relationship to this factor. *Groupements* I and II display the largest loadings (.79 and .80) on factor four, and *groupement* IV the lowest of the class *groupements* (.41).

Thus, the year one complete assessment battery analysis confirms the three-dimensional structure revealed by the total subject sample analysis, with relations *groupements* and memory span variables covarying

TABLE 46

UNROTATED PRINCIPAL COMPONENTS FACTOR MATRIX OF
COMPLETE YEAR ONE ASSESSMENT BATTERY TASKS
(COMBINING KINDERGARTEN AND THIRD GRADE)

	Raw Matrix Factors						
Task	one	two	three	four	five	six	seven
GI	.109	.627	.473	.100	-.035	.214	.051
GII	.023	.664	.462	-.105	-.116	.191	.155
GIII	.083	.407	.250	.039	-.049	-.561	-.301
GIV	.336	.558	-.094	.001	-.365	-.165	-.075
CI	.534	.041	-.208	.166	-.303	-.109	.058
CR	.696	.089	-.055	-.285	-.254	.009	.245
GV	.402	.082	-.244	.422	-.179	.051	-.418
GVI	.641	-.149	.017	.202	-.011	-.040	.054
GVII	.283	.047	-.019	.569	-.016	.545	-.187
GVIII	.546	.025	-.033	.544	.139	-.070	-.112
ASM	.383	.364	-.471	-.267	.507	.183	-.193
AMS	.471	.424	-.373	-.343	.430	.100	-.144
VM	.706	-.079	.024	-.209	-.250	.133	.085
DICH	.691	.081	-.106	.044	-.192	-.017	-.043
SA	.363	-.179	-.158	-.231	-.349	.307	-.159
CARD	.846	.066	-.069	-.143	-.027	-.052	.174
TL	.177	.142	-.061	.408	.230	.174	.575
TW	.264	.150	-.294	.326	.215	-.407	.324
SER	.827	-.080	-.074	-.029	-.097	-.087	-.005
CONNO	.763	-.237	.088	-.077	-.041	.082	.123
IDCONLGTH	.634	-.191	.402	-.068	.283	.016	-.029
EQCONLGTH	.645	-.150	.406	-.182	.240	.016	-.111
IDCONWGT	.593	-.208	.288	.051	.150	-.151	-.179
EQCONWGT	.607	-.119	.395	.120	.203	-.019	-.092
ARITHADD	.849	-.105	-.127	-.109	.010	-.100	.098
Eigenvalues	7.667	1.946	1.704	1.623	1.335	1.145	1.049

TABLE 47

VARIMAX ROTATED FACTOR MATRIX, PRINCIPAL COMPONENTS PROCEDURE, OF COMPLETE YEAR ONE ASSESSMENT BATTERY TASKS (COMBINING KINDERGARTEN AND THIRD GRADE)

	Varimax Factors			
Task	one	two	three	four
GI	.057	-.009	.085	.793
GII	.001	.072	-.118	.804
GIII	.028	.048	.053	.480
GIV	.049	.460	.228	.409
CI	.316	.301	.404	-.060
CR	.569	.498	.033	.059
GV	.137	.172	.597	-.044
GVI	.557	.099	.386	-.074
GVII	.120	-.104	.615	.061
GVIII	.343	.017	.690	.046
ASM	.041	.750	.088	.022
AMS	-.146	.787	.035	.129
VM	.655	.340	.057	-.027
DICH	.494	.376	.332	.035
SA	.333	.274	-.051	-.231
CARD	.673	.496	.213	.044
TL	.008	-.005	.460	.102
TW	-.006	.221	.485	-.028
SER	.687	.372	.287	-.073
CONNO	.762	.190	.154	-.109
IDCONLGTH	.768	-.046	.034	.108
EQCONLGTH	.783	.026	-.057	.140
IDCONWGT	.673	-.053	.153	.030
EQCONWGT	.688	-.104	.201	.168
ARITHADD	.704	.439	.236	-.127
Factor Variance	5.827	2.800	2.445	1.872

with Piagetian tasks, and the first three *groupements* less related to other tasks in the battery. Added to this structure is a large new first factor, consisting of much of the common variance of the additional number-related tasks, defining a dimension strongly related to visual memory. Transitivity of length and weight split off from this factor because of strong format similarity to the relational *groupements* (particularly case V) that define factor three. The full set of Piagetian tasks can thus be located in a space defined by the three dimensions: figural memory, memory span, and the relational *groupements*.

Complete assessment battery analysis--year two data. Table 48 presents the test-retest intercorrelations for the tasks administered to the younger subject subsamples (the complete year one/year two intercorrelation matrix for all inter-task combinations is reported in Appendix A). The correlations range from .144 to .618 with a median value of .423. With the obvious exception of the transitive inference tasks which demonstrated markedly attenuated variability in the second assessment year, the present task array shows a generally satisfactory level of stability.

The year two intercorrelations among the 25 tasks given to the younger subjects during the second assessment year are presented in Table 49. In year two, the full task set displayed similarities to the year one set; principal components analysis again led to seven eigenvalues greater than one, accounting for 64.6% of total variance. The pattern of eigenvalues indicated that five components, accounting for 50.8% of total variance, should be retained. These are given in Table 50. As in year one, the first principal component shows significant common variance among tasks, but fewer extremely high loadings. It is noteworthy that the class *groupements* begin, in this second year, to display some variance in common with the other tasks, and that the transitivity tasks seem to relate differentially to the components of common variance. The second component is again defined by the class *groupements*, with the emergence of significant negative loadings on number conservation and addition, while the third component represents a contrast between the conservation of length and weight tasks and the number-related tasks, a differentiation that was not apparent in year one.

Varimax rotation of the first five principal components gave the results of Table 51. As was the case with year one, the first factor is dominated by the conservation tasks, with class inclusion and combinatorial reasoning showing significant loadings, although in year two it is class inclusion that shows the stronger relation. *Groupement* VII shows a moderate relationship to this "classical Piagetian" factor, joining *groupements* VI and VIII, which retain the relationship observed in year one.

More striking is the set of tasks that are no longer linked to factor one. The number-related tasks, conservation of number, seriation, addition, and cardinality, which figured prominently in the first factor in year one now define factor two, with additional loadings from dichotomies, some-all, *groupement* VI, and visual memory on this "number concept" factor.

TABLE 48

INTERCORRELATIONS BETWEEN INITIAL AND SECOND YEAR PERFORMANCES FOR THE MEASURES ADMINISTERED TO THE KINDERGARTEN/FIRST AND THIRD/FOURTH GRADE SUBJECTS (N = 102)

Tasks	Correlation Values
Groupement I	.230*
Groupement II	.190
Groupement III	.219*
Groupement IV	.411**
Total Class *Groupements*	.441**
Groupement V	.172
Groupement VI	.457**
Groupement VII	.433**
Groupement VIII	.352**
Total Relations *Groupements*	.604**
Class Inclusion	.341**
Combinatorial Reasoning	.322**
Auditory Sequential Memory (Digits)	.536**
Auditory Memory Span (Words)	.550**
Visual Orientation Memory	.593**
Dichotomies	.517**
Some-All	.249*
Cardinality	.578**
Transitivity of Length	.144
Transitivity of Weight	.178
Seriation	.618**
Conservation of Number	.565**
Identity Conservation of Length	.423**
Equivalence Conservation of Length	.358**
Identity Conservation of Weight	.377**
Equivalence Conservation of Weight	.379**
Arithmetic Addition	.446**

* probability value (two-tailed) < .05
** probability value (two-tailed) < .01

TABLE 49

INTERCORRELATIONS AMONG TASKS ADMINISTERED TO YEAR TWO FIRST AND FOURTH GRADE SUBJECT SAMPLE

Task	GI	GII	GIII	GIV	CI	CR	GV	GVI	GVII	GVIII	ASM	AMS	VM
GI	1.000												
GII	.725												
GIII	.494	.507											
GIV	.479	.400	.459										
CI	.426	.249	.293	.293									
CR	.217	.117	.191	.365	.261								
GV	.015	.029	.107	.155	.125	.006							
GVI	.150	-.015	.109	.232	.336	.271	.147						
GVII	.207	.102	.286	.178	.369	.119	.316	.109					
GVIII	.194	.019	.184	.375	.423	.265	.242	.269	.403				
ASM	.277	.243	.309	.379	.289	.302	.068	.159	.245	.320			
AMS	.233	.130	.139	.189	.202	.139	.138	.135	.161	.186	.666		
VM	.379	.273	.203	.335	.325	.324	-.041	.180	.265	.192	.353	.255	
DICH	.165	.095	.242	.271	.510	.278	.121	.270	.313	.304	.167	.246	.306
SA	.063	-.142	.028	.192	.379	.107	.267	.329	.203	.174	.075	.178	.087
CARD	.180	-.007	.058	.292	.296	.254	.140	.292	.184	.325	.219	.178	.390
TL	.240	.165	.157	.156	.283	.138	.114	.219	.304	.128	.147	.102	.151
TW	.161	.102	.105	.183	.108	.143	.194	-.087	.352	.077	.118	.129	.144
SER	.205	.130	.179	.300	.331	.297	.110	.339	.202	.306	.234	.167	.420
CON NO	.045	-.142	.013	.197	.187	.114	.207	.276	.264	.385	.116	.040	.246
ID CON LGTH	.125	.091	.152	.161	.405	.253	.192	.211	.299	.368	.266	.262	.413
EQ CON LGTH	.095	.081	.169	.148	.376	.294	.212	.252	.347	.418	.338	.240	.342
ID CON WGT	.069	-.067	.088	.141	.377	.264	.160	.307	.294	.291	.177	.247	.174
EQ CON WGT	.168	.102	.206	.218	.418	.154	.005	.336	.279	.294	.218	.235	.249
ARITH ADD	.022	-.003	-.180	-.021	.204	.007	.188	.134	.136	.121	.217	.243	.234

TABLE 49, continued.

Task	DICH	SA	CARD	TL	TW	SER	CON NO	ID CON LGTH	EQ CON LGTH	ID CON WGT	EQ CON WGT	ARITH ADD
DICH	1.000											
SA	.366											
CARD	.328	.311										
TL	.289	.290	.104									
TW	.036	-.050	.083	.138								
SER	.517	.174	.496	.190	-.015							
CON NO	.380	.304	.577	.109	.107	.604						
ID CON LGTH	.311	.197	.334	.163	.161	.187	.273					
EQ CON LGTH	.353	.126	.248	.094	.047	.192	.235	.675				
ID CON WGT	.342	.207	.294	.210	.054	.411	.325	.462	.405			
EQ CON WGT	.304	.189	.249	.276	.011	.387	.172	.410	.388	.654		
ARITH ADD	.329	.224	.377	.073	.000	.466	.453	.165	.113	.159	.098	1.000

Note: Value of correlation coefficient significant at the .05 level = .195, and at the .01 level = .254 (two-tailed probabilities).

TABLE 50

UNROTATED PRINCIPAL COMPONENTS FACTOR MATRIX OF
COMPLETE YEAR TWO ASSESSMENT BATTERY TASKS
(COMBINING FIRST AND FOURTH GRADES)

	Raw Factors						
Task	one	two	three	four	five	six	seven
GI	.469	.659	.250	.047	-.113	.032	-.112
GII	.288	.759	.202	.005	-.051	.022	-.220
GIII	.411	.602	-.008	.091	-.191	-.068	.079
GIV	.545	.418	.220	.081	-.067	-.144	.378
CI	.688	.080	-.087	.021	-.254	.118	-.091
CR	.469	.134	.013	-.234	-.007	-.262	.339
GV	.291	-.143	-.166	.625	.111	.085	.231
GVI	.484	-.194	.061	-.138	-.352	.119	.342
GVII	.537	.033	-.297	.467	.053	-.084	-.211
GVIII	.597	-.081	-.160	.113	.037	-.219	.325
ASM	.538	.277	-.046	-.183	.488	.333	.215
AMS	.449	.126	-.103	-.149	.493	.576	.101
VM	.581	.165	.192	-.229	.237	-.180	-.217
DICH	.643	-.164	.133	.028	-.175	.090	-.121
SA	.416	-.319	.070	.275	-.290	.399	.238
CARD	.589	-.296	.349	-.014	.129	-.192	.069
TL	.391	.075	-.055	.252	-.363	.302	-.256
TW	.206	.203	-.172	.497	.321	-.248	-.194
SER	.644	-.230	.465	-.127	-.047	-.114	-.144
CONNO	.531	-.486	.351	.184	.089	-.268	-.008
IDCONLGTH	.618	-.116	-.430	-.142	.154	-.164	-.114
EQCONLGTH	.596	-.097	-.477	-.190	.149	-.168	.024
IDCONWGT	.589	-.292	-.321	-.214	-.171	-.013	-.156
EQCONWGT	.586	-.089	-.292	-.313	-.288	.071	-.240
ARITHADD	.374	-.414	.417	.067	.341	.247	-.273
Eigenvalues	6.644	2.598	1.645	1.462	1.449	1.224	1.127

TABLE 51

VARIMAX ROTATED FACTOR MATRIX, PRINCIPAL COMPONENTS PROCEDURE,
OF COMPLETE YEAR TWO ASSESSMENT BATTERY TASKS
(COMBINING FIRST AND FOURTH GRADES)

	Varimax Factors				
Task	one	two	three	four	five
GI	.090	.082	.837	.114	.051
GII	-.022	-.099	.818	.150	-.009
GIII	.225	-.092	.702	.042	.148
GIV	.157	.231	.649	.126	.126
CI	.564	.266	.357	.019	.189
CR	.342	.181	.277	.247	-.079
GV	.042	.169	-.038	-.040	.709
GVI	.489	.377	.110	-.142	-.074
GVII	.317	.114	.158	.095	.672
GVIII	.416	.271	.107	.187	.332
ASM	.166	.135	.296	.704	.105
AMS	.151	.128	.120	.650	.133
VM	.200	.357	.382	.465	-.056
DICH	.412	.516	.194	-.007	.126
SA	.299	.440	.003	-.282	.278
CARD	.160	.708	.066	.194	.064
TL	.336	.138	.293	-.255	.273
TW	-.106	-.056	.158	.220	.613
SER	.233	.765	.210	.096	-.097
CONNO	.103	.786	-.088	.036	.214
IDCONLGTH	.616	.112	-.051	.418	.235
EQCONLGTH	.643	.056	-.062	.435	.202
IDCONWGT	.643	-.062	.379	.216	.052
EQCONWGT	.762	.138	.090	.079	-.062
ARITHADD	-.111	.712	-.131	.246	.095
Factor Variance	3.396	3.332	3.145	2.032	1.897

The class *groupements* define factor three, but as was not the case in the previous year's class factor (four), certain other tasks begin to show a relationship to the class *groupements* factor. In particular, identity conservation of weight, class inclusion, and visual memory display loadings of over .35, and combinatorial reasoning, transitivity of length, and sequential memory have loadings approaching .30.

Factor four is a memory factor, comparable to factor two in year one. The two length conservation tasks also display moderate loadings on this factor, as was not the case in year one. Finally, factor five represents a splitting up of the relational *groupements*, with *groupements* V and VII and transitivity of weight showing loadings of above .60, *groupement* VIII a loading of .33, and *groupement* VI an essentially zero loading.

The splitting up of the transitivity tasks, with transitivity of length showing a greater degree of relationship to conservation tasks and *groupements* VI and VIII, while transitivity of weight shows a greater degree of association to *groupements* V and VII, is more likely to be due to differential ceiling effects than to a content distinction, since *groupements* V-VIII all involve both length and weight. The overriding distinction between the two sets of relational *groupements* is due to elements of transitivity of equivalence in *groupements* VI and VIII versus elements of transitivity of nonequivalence (e.g., shorter, heavier) in *groupements* V and VII. If transitivity of weight indeed depends more on the ability to handle inequality than does transitivity of length, it would seem that differences in ease of encoding the two dimensions might be involved. If replicated, the opposite loadings of the two transitivity tasks on the memory factor would also require explanation.

Thus, the complete assessment battery analysis for year two shows evidence of increasing differentiation among tasks, with the emergence, in factor three, of a relationship between the class *groupements* and a limited set of Piagetian tasks. The more perceptual memory span tasks become less implicated in the performance of other tasks, but the more conceptual memory for orientation becomes less implicated in conservation and more in the classification tasks, while retaining its connection to number-related performance. Again in year two the complete assessment battery analysis is consistent with the total subject sample analysis, but reveals complexity in development of certain skills that require a more detailed task analysis for explanation.

The general but differential involvement of memory in these tasks suggests the need for studies in which modality and conceptual load are both studied at each level of complexity, and in which a thorough performance analysis, including memory load and dimensional salience, is carried out to complement the logical task analysis traditional in Piagetian research. The pervasive role of visual memory in these analyses suggests that it is a variable that must be carefully taken into account if we are to identify the deeper structural elements from among the several factors influencing surface performance and to fully understand the complex interrelationships that go into the development of these abilities.

IV

DISCUSSION

As the preliminary considerations section indicates, the present array of Piagetian concrete operations tasks offers an acceptable level of internal consistency and is relatively free of pronounced order-of-presentation influences. As anticipated, the initial year's data analyses revealed notable grade level main effects for all the tasks (excluding the length transitive inference case). With the exception of the observed female class inclusion task performance superiority, significant sex main effects or higher order interactions were generally absent. The relative order of difficulty for the present task series is in essential accord with the preceding investigations which have utilized similar task formats. Examples of significant differential inter-task performance comparisons include the following: the greater difficulty of class inclusion tasks contrasted with some-all understanding (cf. Hooper, Sipple, Goldman, & Swinton, 1974; Kofsky, 1966); the greater difficulty of conservation tasks contrasted with content dimension matched transitive inference task formats (cf. Brainerd, 1973b; Peterson, Hooper, Wanska, & DeFrain, 1976); the greater difficulty of equivalence conservation tasks contrasted with their identity conservation counterparts (cf. Brainerd & Hooper, 1975); and the greater difficulty of the classificatory *groupement* cases contrasted with the relational *groupements* (cf. Brainerd, 1972). The results for the subset of measures dealing with number concept understanding are less distinct and appear to be dependent upon the task and scoring criteria selected (Gonchar, 1975).

The second assessment year's data analyses essentially corroborate the generalizations drawn from the initial cross-sectional results. No consistent significant distinctions were found between the continuing longitudinal sample participants and the nonparticipant subsamples initial year's performances. In addition, the comparisons between the longitudinal and testing control subsamples second year assessments indicated few consistent or pervasive influences attributable to repeated testing effects (see Tables 21 to 25). Grade level comparisons were once again highly consistent and uniformly significant, while sex differences were notably absent except for the class inclusion female superiority case cited above (see Tables 26 to 31). As anticipated, year-of-measurement effects (longitudinal change indices) were significant for all the logical reasoning tasks except the transitive inference cases and some-all understanding (ceiling effects were clearly in

evidence for these measures). A considerable number of significant grade level/assessment year interactions was observed apparently following from the differential item difficulties and nonconcordant growth rates for the present age-grade subsamples. Finally tentative evidence was presented for a developmental sequence between two distinct logical concept domains, transitive inference → conservation, which is not compatible with orthodox Piagetian structural theory. Conservation tasks were consistently more difficult than transitive inference tasks (equated for content dimensions) and these differences were most pronounced for the younger subjects (whether viewed in terms of grade level or assessment year). Pass/fail comparisons of the second assessment year's performances of subjects who passed transitivity while failing the counterpart conservation task in the initial year's assessments tended to substantiate this proposed developmental sequence (cf. Hooper & Toniolo, 1976). As may be expected, evidence was also presented for a transitive inference → class inclusion sequence. However, there were no significant distinctions between subjects' conservation and class inclusion performances, contrary to the conclusions of Brainerd (1973b). These comparisons may be confounded with other sources of performance variance, however, since the present class inclusion task format utilized two-dimensional geometric forms unlike either the transitive inference or conservation task settings.

We may now turn to the implications of the correlational results and related dimensional analyses. The present report has examined on an exploratory basis the relationships among a battery of Piagetian concrete operations tasks, a set of eight cognitive tasks designed to directly assess the *groupements* postulated by Piaget to form the basis of concrete operational reasoning (cf. Brainerd, 1972), and measures of short-term auditory span and visual orientation memory. These data, collected in the initial two years of a continuing longitudinal study (cf. Hooper & Klausmeier, 1973), make it possible to delineate relationships on the full task set from kindergarten to grade one and from grade three to grade four. A restricted subset of tasks, appropriate to the age level, was also examined from grade six to grade seven. Thus it was possible to study the relative involvement of the class and relational *groupements* and memory tasks with the classical Piagetian tasks both cross-sectionally and longitudinally.

In the total subject sample analysis, including all subjects through grades six and seven in years one and two, respectively, classical Piagetian measures were represented only by class inclusion and combinatorial reasoning. The wide age range increased the likelihood that the 13 tasks would contain a strong general factor, and previous work, as well as theoretical considerations, predicted that the class and relational *groupements* would form two more-or-less distinct dimensions.

The questions of interest in this analysis were thus the relationship of the memory tasks to the *groupements*, and the location of the Piagetian tasks in the space determined by the *groupements* and memory. Three principal components, accounting for 57% of the total variance, were extracted from the matrix of correlations of the year one task assessments. Varimax rotation led to a first factor determined by

the relational *groupements*, with significant loadings on both Piagetian tasks and visual memory. The second factor consisted of the four class *groupements* with no loadings above .30 on any other tasks. The behavior of class *groupement* IV, showing the smallest loading on the class *groupement* factor, and displaying a moderate loading on the relational and Piagetian factor, led us to question the validity of the set of measures intended to operationalize the class *groupement* construct. The third factor, determined by the two auditory memory tests, was similar to factor one in its pattern of relations to the Piagetian tasks and to visual memory, although while class inclusion showed a relatively stronger dependency on the relational *groupements*, combinatorial reasoning seemed to depend somewhat more strongly on the memory factor.

The correlations among tasks in the following year were also subjected to principal components analysis. Varimax rotation of the first three components, accounting for 55% of the total variance, again led to factors determined by relational *groupements*, class *groupements*, and memory. However, for the year two class *groupements* factor, a relationship emerges with class inclusion and visual memory, in contrast to the isolation of this factor from other tasks in year one.

Analysis of the year one complete assessment battery matrix (grades K and 3) confirms the three dimensions: relational *groupements*, class *groupements*, and auditory memory span, revealed in the full sample subset of year one tasks. Again, the first three class *groupements* determine a dimension unrelated to the variance of the other tasks. The large additional set of number-related Piagetian tasks, conservation, addition, and visual memory, cluster together to determine a new dimension of variance, relating to the two Piagetian tasks of the total subject sample analysis, and leading to a representation of the covariance of the concrete operations tasks in a three-dimensional space which may be defined by the relations *groupements*, auditory memory span, and visual, or figurative memory. Again, the class *groupements* do not appear to be needed to describe the underlying structure of the year one concrete operations tasks.

As had been the case in the year two total subject sample analysis, the class *groupements* show certain relationships to Piagetian tasks in the year two complete task battery (grades 1 and 4). In addition to class inclusion and visual memory, combinatorial reasoning, transitivity of length, identity conservation of weight and auditory sequential memory show moderate relationships to the class *groupements* in this more extensive battery. With the larger number of tasks and more restricted age range, the year two complete task analysis exhibits a factorial complexity that raises new questions concerning performance variation in the tasks. While the analysis has established that memory span and the more conceptual visual memory for orientation play an important role in performance on concrete operational tasks, the issue of direction of causality cannot be resolved in a correlational study. Such techniques as cross-lag correlational analysis (cf. Wohlwill, 1973) are possible in this data set, but interpretation is complicated by the apparent change in meaning of tasks over time. Consider, for discussion purposes, orientation memory and combinatorial reasoning correlation patterns from year one to year two (see Figure 3).

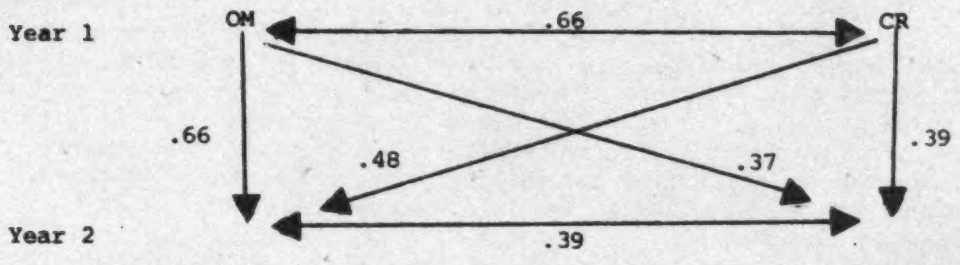

Figure 3. Cross-lag year one and year two correlations for the orientation memory and combinatorial reasoning tasks.

Given the different stability of the tasks over the two-year period, related to the changing dependence of CR on the class *groupements* in year 2, how should we appropriately weight the facts that while combinatorial reasoning (CR) predicts memory (OM) relatively better than OM predicts CR, on the other hand OM predicts CR nearly as well as CR predicts itself? Clearly some other causal influence is operating on CR, underscoring the necessity of operating in a closed system when causal inferences are to be drawn.

Kenny (1973, 1975) suggests a method of correcting cross-lag correlations for changes in reliability over time, if the assumptions of synchronicity and quasi-stationarity are met. He warns, however, that a change in stage may imply that the causal structure itself changes over time, making quasi-stationarity less plausible during periods of rapid growth.

Applying Kenny's procedure to the system of four variables: orientation memory (OM), combinatorial reasoning (CR), class *groupement* total (CG), and relational *groupement* total (RG), each measured at two points in time (see Table 52), we obtain the corrected correlations of Figure 4.

The corrected synchronous correlations, .500 and .517, are nearly equal, suggesting that the assumption of quasi-stationarity may be tenable. The direction of the corrected cross-lag differential suggests that changes in classificatory reasoning ability are prior to changes in orientation memory: that is, conceptual memory is better viewed as a product of the developed logical structure than as a cause of the development of that structure. This result is in accordance with the

TABLE 52

CORRELATIONS AND RELIABILITY RATIOS FOR
CROSS-LAG ANALYSIS BY TASK AND YEAR OF ASSESSMENT

	OM1	CG1	RG1	CR1	OM2	CG2	RG2
CG1	.447						
RG1	.417	.365					
CG1	.663	.452	.366				
OM2	.660	.355	.445	.478			
CG2	.346	.595	.362	.360	.415		
RG2	.417	.268	.595	.343	.305	.407	
CR2	.360	.340	.407	.389	.390	.361	.343

Pooled K^2_{OM} = .580 K_{OM} = .762

Pooled K^2_{CR} = .638 K_{CR} = .799

Pooled K^2_{RG} = 1.100 K_{RG} = 1.049

Pooled K^2_{CG} = 1.182 K_{CR} = 1.087

Piagetian view of memory as an active cognitive restructuring rather than merely a passive capacity.[2]

It is clear that the present dimensional analyses, whether viewed in terms of our general inferences concerning Piaget's contentions (cf. Piaget, 1972; Pinard & Laurendeau, 1969; Wohlwill, 1973) or in terms of Brainerd's specific predictions (cf. Brainerd, 1972), present a decidedly non-unitary picture. Each individual year's analyses revealed a complex factor structure and this was true for the total subject sample and younger subsample cases. While the orthodox Piagetian assumption of structural simplicity was not born out, neither was Brainerd's (1972) prediction of discrete class and relational factors unequivocally demonstrated. Moreover, the factor patterns alter from the initial to the second assessment year. It can only be concluded that, on the basis of the present correlational and task mastery sequence results, the best characterization of the concrete operations period is a complex pattern of interrelationships which is decidedly multidimensional in nature. It also appears that Brainerd's (1972) array of operationally defined logical *groupement* tasks do not subsume all of the performance variations found with more conventional Piagetian concrete operations task formats.

If the present two year findings are substantiated in the extended longitudinal analyses to follow, the general developmental construct of the concrete operations period of the middle-childhood years will have to undergo considerable revision. The transition from preoperational forms of logical reasoning to a full mastery of the concepts derived from the logical *groupements* appears not to be a punctate, discontinuous

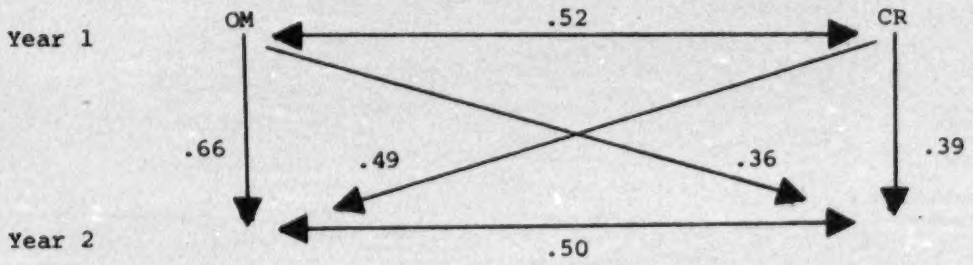

Figure 4. Corrected year one and year two correlations for the orientation memory and combinatorial reasoning tasks.

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[2]While suggestive, this asymmetry fails to reach significance when tested by the Pearson-Filon test (Kenny, 1975, p. 896).

$$z = \frac{\sqrt{156}\ (.489 - .357)}{\sqrt{(1 - .489^2)^2 + (1 - .357^2)^2 - .321}} = 1.618 \quad p < .11$$

If the corrected correlations remained the same, a sample size twice that used in this study would lead to a rejection of this null hypothesis at the .05 level.

phenomenon with uniform progress being demonstrated for all the various task settings. Rather, the acquisition of concrete operations is an extended developmental process marked by unevenness and "reciprocal interweaving" among the putatively unitary logical concept domains (cf. Flavell, 1971; Wohlwill, 1973). As a single exemplar, mastery of addition and multiplication of relations (e.g., transitive inference, serial ordering, and the four relational *groupements*) appears to be a significantly earlier developmental achievement in contrast to certain counterpart classificatory tasks (e.g., class inclusion and the four classificatory *groupements*). Yet class inclusion, as assessed in this battery, demonstrated latent links to the relational *groupements* tasks.

Short-term memory span was operationalized in this study as a perceptual-level forward repetition of strings of familiar words and digits, and as a more conceptual visual recognition of the orientation of abstract forms. Reese (1973) presents a list of 10 dimensions on which memory can be characterized, pointing out that the list could go on, but that the point has already been made that memory is an extremely complex topic. Just as any operationalization of the concrete operations tasks must introduce sources of variance irrelevant to the construct being measured, so for memory. In addition to the obviously differing visual perceptual demands of the two memory formats employed, vocabulary, knowledge of left and right, and willingness to guess in the multiple-choice visual memory format make these tasks less than pure measures of memory. At the very minimum, both modalities should be represented at both conceptual levels, as should both recognitive and reconstructive memory. Already this suggests considering eight memory tasks, and we have not begun to cover Reese's admittedly incomplete list. Thus not only is a multi-trait, multi-method approach (cf. Campbell & Fisk, 1959) indicated, but also an extreme precision of variable specification that does not gratuitously assume that two even slightly different procedures tap the same process. Such precision, characteristic of research in verbal learning, largely has been honored in the breach in studies of perceptual-cognitive development. The task of expanding Reese's list into a taxonomy of dimensions on which "memory" varies is a laborious but necessary step before summaries of existing research and plans for future studies are likely to lead to cumulative knowledge in this area.

While this study has demonstrated that memory variables should be taken into account, the splitting off of number-related tasks and visual memory from conservation, class inclusion, combinatorial reasoning and the relational *groupements*, the separation of transitivity of weight from transitivity of length, and the splitting up of the relational *groupements* themselves in year two, suggest that further task analyses are needed. Other important sources of performance and content variance, not included in Piaget's logical structures nor in the memory battery administered in the present study, must be identified and measured before the underlying structural relations of the concrete operations period can be accurately mapped.

Particular attention should be paid to developing alternative versions of tasks embodying a given logical structure, but varying systematically on such performance factors as memory load, visualization requirements, stimulus salience, and verbal demands, to make possible the assessment of the relative contribution of these factors to task difficulty, and to reduce the impact of ceiling and floor effects on studies of interelationships of underlying constructs.

Until the factors contributing to performance variation are more fully understood and controlled, Piagetian research will be in the position of attempting to study relationships among unobservable phenomena with measures that indirectly reflect those phenomena, but that also reflect other unknown sources of difficulty and covariation to unknown and differing degrees. While global description is possible under these circumstances, the precision needed for causal inference and hypothesis testing concerning logical structures requires psychometric sharpening of our tools.

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APPENDIX A

INTERCORRELATIONS AMONG INITIAL AND SECOND YEAR PERFORMANCES

TABLE A-1

INTERCORRELATIONS AMONG INITIAL AND SECOND YEAR PERFORMANCES FOR THE MEASURES ADMINISTERED TO THE TOTAL SUBJECT SAMPLE (N = 156)

Task / Year 1 / Year 2	GI	GII	GIII	GIV	GI-IV	CI	CR	GV	GVI	GVII	GVIII	GV-VIII	ASM	AMS	VM
GI	.424	.395	.134	.507	.478	.366	.268	.187	.233	.214	.233	.306	.354	.253	.322
GII	.431	.413	.180	.459	.483	.256	.210	.143	.203	.172	.224	.262	.323	.291	.267
GIII	.370	.337	.343	.391	.481	.207	.281	.112	.166	.265	.280	.302	.258	.278	.164
GIV	.342	.271	.271	.505	.471	.272	.367	.154	.198	.195	.271	.291	.364	.386	.364
GI-IV	.479	.430	.305	.577	.595	.335	.360	.182	.245	.267	.319	.362	.402	.384	.346
CI	.231	.283	.152	.456	.380	.416	.371	.349	.376	.286	.355	.476	.240	.233	.388
CR	.206	.102	.292	.367	.340	.332	.389	.260	.337	.195	.381	.407	.212	.263	.366
GV	.088	.070	.137	.023	.105	.119	.088	.179	.192	-.021	.307	.217	.128	.182	.168
GVI	.139	-.001	.063	.229	.151	.210	.320	.192	.484	.264	.326	.439	.200	.176	.318
GVII	.060	.106	.053	.282	.177	.238	.213	.255	.312	.368	.362	.464	.151	.267	.243
GVIII	.167	.189	.144	.303	.273	.298	.303	.300	.341	.276	.396	.460	.233	.280	.394
GV-VIII	.164	.143	.143	.325	.268	.328	.343	.348	.486	.350	.516	.595	.262	.342	.417
ASM	.091	.110	.105	.209	.178	.179	.129	.212	.224	.285	.274	.356	.506	.465	.181
AMS	.211	.256	.089	.227	.253	.273	.239	.200	.160	.221	.263	.302	.545	.580	.361
VM	.317	.261	.148	.353	.355	.364	.478	.254	.358	.272	.383	.445	.351	.318	.660

TABLE A-2

INTERCORRELATIONS AMONG INITIAL AND SECOND YEAR
PERFORMANCES FOR THE COMPLETE ASSESSMENT BATTERY TASKS (N = 102)

Task: Year 1 / Year 2	GI	GII	GIII	GIV	GI-IV	CI	CR	GV	GVI	GVII	GVIII	GV-VIII	ASM	AMS	VM
GI	.230	.187	-.019	.420	.311	.269	.164	.224	.156	.195	.135	.254	.196	.187	.169
GII	.238	.190	.059	.389	.334	.057	.055	.118	.132	.100	.070	.147	.099	.157	.026
GIII	.253	.201	.219	.317	.377	.018	.205	.165	.154	.280	.204	.299	.131	.173	-.025
GIV	.176	.033	.248	.411	.357	.152	.327	.109	.169	.134	.203	.221	.320	.405	.237
GI-IV	.280	.181	.189	.482	.441	.148	.259	.189	.195	.229	.206	.297	.248	.310	.133
CI	.055	.138	.058	.357	.249	.341	.356	.431	.438	.294	.360	.531	.170	.163	.302
CR	.110	-.036	.150	.271	.210	.238	.322	.292	.308	.133	.344	.370	.105	.135	.274
GV	-.032	-.070	.129	-.080	-.012	.062	.009	.172	.151	-.077	.299	.172	.048	.110	.055
GVI	.030	-.149	-.038	.140	.009	.128	.273	.210	.457	.241	.302	.426	.152	.130	.277
GVII	.010	.026	-.043	.270	.117	.179	.198	.343	.310	.433	.383	.535	.160	.239	.282
GVIII	.031	.065	.069	.180	.142	.208	.227	.345	.285	.227	.352	.424	.146	.251	.298
GV-VIII	.017	-.039	.038	.211	.106	.225	.275	.414	.457	.335	.507	.604	.196	.284	.356
ASM	.064	.032	.106	.166	.151	.149	.146	.254	.221	.285	.316	.390	.536	.499	.178
AMS	.035	.069	-.034	.108	.067	.185	.129	.176	.150	.156	.221	.251	.533	.550	.236
VM	.135	.108	-.055	.229	.155	.238	.323	.250	.332	.191	.263	.361	.275	.266	.593

TABLE A-2, continued.

Task / Year 1 / Year 2	LICH	SA	CARD	TL	TW	SER	CONNO	IDCON LGTH	EQCON LGTH	IDCON WGT	EQCON WGT	ARITH ADD
GI	.117	.169	.174	.251	.091	.203	.166	.260	.321	.187	.257	.205
GII	.099	.075	.060	.090	.078	.091	.069	.188	.278	.043	.124	.085
GIII	.083	.108	.135	.169	-.007	.078	.163	.226	.245	.050	.181	.048
GIV	.261	.175	.407	.122	.086	.310	.362	.286	.316	.157	.190	.270
GI-IV	.202	.170	.268	.195	.074	.224	.261	.308	.365	.137	.237	.197
CI	.285	.221	.491	.163	.174	.465	.406	.254	.337	.382	.348	.389
CR	.358	-.029	.403	.157	.134	.345	.351	.170	.288	.163	.369	.395
GV	.094	.034	.104	.076	.321	.254	.105	.216	.177	.130	.174	.113
GVI	.297	-.034	.366	.033	.010	.256	.277	.233	.287	.160	.245	.283
GVII	.202	.169	.258	.345	.264	.297	.232	.154	.142	.140	.209	.257
GVIII	.259	.240	.373	.223	.114	.372	.384	.169	.181	.338	.320	.343
GV-VIII	.326	.167	.423	.271	.264	.449	.385	.285	.292	.295	.361	.384
ASM	.182	.158	.227	.185	.154	.222	.246	.269	.368	.261	.264	.293
AMS	.170	.108	.234	-.014	.205	.226	.155	.210	.296	.246	.282	.317
VM	.442	.117	.480	.234	.167	.406	.402	.404	.504	.243	.259	.427

TABLE A-2, continued.

Task / Year 1 / Year 2	GI	GII	GIII	GIV	GI-IV	CI	CR	GV	GVI	GVII	GVIII	GV-VIII	ASM	AMS	VM
DICH	-.034	.114	.032	.259	.159	.265	.481	.218	.533	.143	.387	.439	.258	.310	.474
SA	-.156	-.042	.019	.125	.007	.338	.329	.123	.209	.163	.177	.241	.234	.266	.321
CARD	.022	.043	.053	.095	.087	.312	.305	.168	.270	.120	.378	.327	.249	.309	.431
TL	-.058	-.196	-.108	.199	-.035	.325	.219	.021	.260	.184	.233	.256	-.011	.151	.197
TW	.188	.016	.095	.160	.174	.083	-.003	.079	.056	.117	.138	.145	.089	.202	.055
SER	.100	.133	.137	.173	.209	.194	.410	.183	.393	.135	.339	.363	.295	.275	.436
CONNO	-.078	-.009	.042	-.022	-.017	.278	.386	.021	.311	.006	.310	.219	.242	.262	.462
IDCONLGTH	-.020	.028	.113	.181	.135	.216	.309	.370	.399	.238	.268	.441	.044	.087	.448
EQCONLGTH	-.081	.088	.104	.204	.143	.197	.255	.324	.410	.174	.305	.416	.023	.084	.391
IDCONWGT	.088	.053	.078	.046	.097	.137	.184	.202	.329	.182	.305	.358	.214	.141	.355
EQCONWGT	.093	.065	.072	.178	.160	.218	.201	.324	.326	.244	.233	.395	.144	.169	.263
ARITH ADD	-.113	.027	-.041	-.020	-.051	.383	.293	.174	.311	.000	.310	.263	.291	.292	.290

TABLE A-2, continued.

Task: Year 1 / Year 2	DICH	SA	CARD	TL	TW	SER	CONNO	IDCON LGTH	EQCON LGTH	IDCON WGT	EQCON WGT	ARITH ADD
DICH	.517	.222	.599	.096	.189	.553	.526	.446	.413	.381	.432	.586
SA	.315	.249	.320	-.019	.192	.428	.279	.177	.150	.216	.255	.373
CARD	.450	.123	.578	.057	.147	.492	.532	.359	.319	.496	.382	.461
TL	.155	.066	.257	.144	.149	.287	.262	.187	.130	.162	.242	.396
TW	.044	-.072	.197	.269	.178	.110	.077	.035	.096	.051	.151	.132
SER	.530	.042	.614	.216	.204	.618	.641	.445	.480	.451	.407	.533
CONNO	.413	.160	.501	.192	.259	.583	.565	.352	.294	.399	.224	.516
IDCONLGTH	.330	.169	.379	.030	.182	.403	.399	.423	.354	.387	.338	.348
EQCONLGTH	.241	.158	.287	.102	.172	.379	.319	.296	.358	.327	.280	.298
IDCONWGT	.320	.040	.404	-.023	.174	.409	.414	.312	.224	.377	.406	.341
EQCONWGT	.315	.252	.338	.100	.173	.345	.403	.322	.286	.403	.379	.327
ARITHADD	.383	.096	.398	.161	.398	.525	.448	.368	.279	.282	.207	.446

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